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THEORY OF THE ETHER DRIFT MEASUREMENT USING A FORTY-FIVE DEGREE INTERFEROMETER

by

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INTRODUCTION

The postulate of a luminiferous ether, in which all matter is immersed, is quite generally accepted by physicists as the most satisfactory hypothesis upon which to base an explanation of the phenomena of radiation. The ether drift experiment was originally conceived as a means of experimentally testing, and, if possible, verifying this hypothesis. James Clerk Maxwell was probably the father of the idea, for it was an article which he wrote for the 1878 edition of the Encyclopedia Britannica which first called attention to the effect which this experiment endeavors to measure. The article pointed out that if there does exist a stationary ether in which the earth is moving with some absolute velocity, and if light is transmitted through this medium in the form of waves or vibrations, then due to the time required for transmission, a path in the direction of motion would be slightly lengthened as compared with a similar path perpendicular to the motion. However, since any attempt to measure the difference between these two paths must require the light to pass from the source to the end of the path and back again to the source, the shortening of the return path would so nearly counteract the increase of the forward path that he concluded: "The change in the time of transmission of the light on account of a relative velocity of the ether equal to that of the earth in its orbit would be only one hundred-millionth part of the whole time of transmission and would, therefore, be quite insensible."

In 1880, while a student at the University of Berlin, the late Professor Albert A. Michelson devised the instrument now known as the "Michelson Interferometer" as a means of experimentally testing the stationary ether hypothesis. By means of this device a beam of light, falling upon a half silvered plate, is divided so that each half proceeds along a separate path in a different direction. At the end of each path, each beam is reflected and returned to the source where the two beams are again united and sent out in directions almost parallel. As a result, the wave fronts of the reunited rays interfere, alternately re-enforcing and destroying each other and setting up interference fringes. As the instrument is rotated, first one arm has the longer effective path and then the other; so that these interference fringes, which are actually due to the phase relationships between the wave fronts of the reunited rays, will undergo slight shifts, the relation being such that an effective change of one wave length in the length of one path as compared with the other will result in a movement of the fringes equal to one fringe width. Of course, this effect is dependent entirely upon the assumed fact that the instrument is moving in a stationary medium, and that the light is being transmitted in this stationary medium entirely independent of the motion of the source. If the stationary ether hypothesis is not correct, no displacement of the fringes should take place.

Michelson, and later Michelson assisted by Professor

Edward W. Morley of Western Reserve University, as well as several other scientists, have at various times constructed interferometers, and conducted careful experiments for this expected fringe shift effect. Professor Dayton C. Miller, in an article which appeared in the July, 1933, issue of the Reviews of Modern Physics presents a summary of the results of some of these investigators, as well as an exhaustive analysis of very extensive investigations of his own. The conclusion reached by Professor Miller, and at present undisputed among physicists, is that a small, actual, periodic fringe shift is obtained, but that it is entirely too small to be accounted for on the basis of the stationary ether hypothesis. The orbital velocity of the earth has long been known, and the actual fringe shifts obtained were much too small to account for a relative velocity between the earth and the ether of even this amount.

The earliest work in this experiment, carried out by Michelson in 1881 and repeated in 1887, had also indicated an almost null effect so that physicists set themselves to the task of discovering a suitable explanation. In 1891, Professor Fitz Gerald of Dublin offered the theory that something inherent in motion itself causes the particles of a solid to shrink together in the direction of motion just enough to offset the expected increase in the length of path, thereby destroying the expected effect. In 1895, Professor Lorentz of Leyden put this theory on a firm basis mathematically, and showed how it is possible that the one effect exactly offsets the other.

The next development of this attempt to postulate conditions which would lead to a null effect of the ether drift came in 1905 when Professor Albert Einstein announced his Special Theory of Relativity. This also leads to a shortening of a material body in the direction of motion, resulting in a null effect of the ether drift. However, in view of the definite, positive effect recorded by the exhaustive work of D. C. Miller, even though this effect was small, an hypothesis which leads us to expect a null effect is not much more comforting than the original stationary ether hypothesis. It is quite satisfying, then, to examine the broader and more flexible postulatesof Professor Einstein's General Theory of Relativity announced in 1915 and to note that they may be modified in such a way as to include any relative velocity of drift. Certainly such achievements of the General Theory as accounting for the advance in the perihelion of mercury, predicting the bending of a ray of light when it passes through a strong gravitational field and predicting that when the source of light is in the atmosphere of a massive star the wave length becomes slightly more, are of such consequence as to lend great support to the General Theory. The Special Theory, on the other hand, with its prediction of a null effect to the ether drift experiment, may only be an approximation of the General Theory, and thus not actually be in violation of it.

It is the hope of the author that this paper may prove of help in answering the ether drift question. The original

experiments of Michelson and Morley, as well as the more recent extensive experiments of D. C. Miller, were all carried out by means of a rectangular interferometer, that is, one in which the divided beam followed paths perpendicular to each other. Early last fall the writer and Dr. Royal M. Frye attended the meeting of the American Physical Society at Wellesley College where Mr. W. B. Cartmel presented a brief paper claiming that for the greatest fringe shift effect the arms of an interferometer should be inclined forty-five degrees to each other. Mr. Cartmel gave a formula, discussed briefly in another part of this paper, in which the fringe shift is given as a function of the angle between the arms as well as the orientation with respect to the direction of drift. At the suggestion of Dr. Frye, certain checks were applied to Cartmel's formula, and in at least one respect the formula looked very unreasonable. However, the author decided to investigate the problem himself, and the work contained in this paper is the result.

A brief discussion of Cartmel's work is included in the conclusion. Qualitatively, the writer agrees with him, but the form of the author's equation of shift is quite different. In fact, it requires an even greater fringe shift than does the formula given by Cartmel. Compared to the fringe shift which may be expected from a rectangular interferometer, the writer has shown that an interferometer with arms inclined at forty-five degrees should show a shift over twice as great.

The foregoing remarks are intended to make clear the function of this paper. Certainly, no satisfactory theory of the nature of the ether is tenable until the question of the ether drift is settled satisfactorily, and if the looked-for effect of the ether drift experiment is more than doubled by simply changing the arms of the interferometer to meet at forty-five degrees, it would seem as if anyone undertaking the experimental investigation of this problem would want to take advantage of this fact.

REVIEW OF THE WORK OF OTHERS

The ether-drift experiment has a long and interesting history. As early as 1880 and 1881, A. A. Michelson, while a student in Germany, conducted experiments at both Berlin and Potsdam. These were the first experiments ever undertaken by anyone for the express purpose of detecting an effect of the second order due to a relative velocity between the earth and the ether. A second order effect is one proportional to the square of the ratio of the velocity of the earth relative to the ether. Previous attempts had been made, without success, to detect a first order effect.

These earliest attempts of Michelson[9] were also without success, but this is not surprising when we learn that he was using, at that time, an interferometer with a light path only a little more than one meter in length. Upon his return to America, Professor Michelson interested E. W. Morley, of the Case School of Applied Science, in the experiment, and together they repeated the investigation. This was in 1887. The base of the interferometer used in these experiments was a solid block of sandstone, and by means of repeated reflections of the beams, the light path was increased to something over ten meters in length. A small but positive effect was noted and reported[10], but it was less than five per cent of the

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[9] A. A. Michelson, Am. J. Sci., 22, 120 (1881)

[10] A. A. Michelson and E. W. Morley, Phil. Mag., 24, 449, (1887)

magnitude expected.

In 1900, E. W. Morley and D. C. Miller were present in Paris at the International Congress of Physics where Lord Kelvin expressed the conviction that the ether drift experiment should be repeated. Accordingly, the two constructed an interferometer having a light path of about 214 feet. They first used a framework of heavy lumber, but this was soon replaced by one made of structural steel and floated in a tank of mercury. Between 1900 and 1907[12] many observations were made, some in the basement of the Case School and later at a site on Euclid Heights, Cleveland, which was in comparatively open country.

About this time Professor Morley retired from active work and nothing more was done for a number of years. In 1921, however, D. C. Miller resumed the investigation alone, and the ether drift interferometer was set up at the observatory on top of Mount Wilson. Here the elevation was over six thousand feet. A series of observations were made at Mount Wilson, and then the interferometer was returned to the laboratory in Cleveland, where many trials were made during 1922 and 1923. In 1924, it was returned again to Mount Wilson where further observations were made, this time with improved temperature controls.

Altogether, over one hundred thousand readings were taken with this instrument, representing the most exhaustive study in this field. The analysis and summary of the results of this

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[12] E. W. Morley and D. C. Miller, Phil. Mag., 9, 680, (1905)

investigation are reported at some length in the Reviews of Modern Physics[11], together with a fairly complete history of the efforts of D. C. Miller. The conclusion reached by Miller is that there exists a small positive fringe shift, equal in magnitude, however, to only about five per cent of the expected effect.

Other investigators have also studied the ether drift problem. R. J. Kennedy[7] and E. M. Thorndike carried out experiments on an interferometer of quite a different design. The arms were purposely made very unequal in length, and were not perpendicular, but were inclined at such an angle that the light reflected from the plate would be plane polarized. However, a careful examination of the report of this experiment fails to show that the conductors had any idea that the effect for which they were looking would be increased by having the arms meet at this angle of about sixty-five degrees. They developed a theory based on the Lorentz - Fitz Gerald contractions and showing that no effect should be expected. If the Lorentz - Fitz Gerald contractions be assumed, the effect of the ether drift is null, and independent of the angle between the arms of the interferometer. The length of the light path used was comparatively short, and the conductors reported only an extremely small effect, if any.

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[11] Dayton C. Miller, Rev. Mod. Phys., 5, 203, (1933)

[7] R. J. Kennedy and E. M. Thorndike, Phys. Rev., 42, 400, (1932)

In 1927, A. Piccard[13,14] and R. Stahel reported small effects somewhat in agreement with the results of D. C. Miller, while Georg Joos[6] in 1930 conducted a number of experiments on a rectangular interferometer and reported no satisfactory evidence of an effect.

The theoretical side of the ether drift experiment has been treated by a number of writers. W. M. Hicks[5] of the University College of Sheffield, England, wrote a lengthy discussion of it which appeared in the Philosophical Magazine in 1902. Hicks considered especially the effect produced in a rectangular interferometer when the mirrors are not exactly perpendicular to the arms. According to him, this introduces a small first order effect which is singly periodic in a complete rotation of the interferometer. This effect, however, is variable with the width between fringes, and although Miller believes this effect was present in his observations, he made no attempt to analyze or study it. Miller's entire analysis was of the second order doubly periodic effect, determined from his readings by means of the harmonic analyzer.

H. A. Lorentz[8], in his book "The Theory of Electrons"

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13 A. Piccard and R. Stahel, Comptes Rendus, 183, 420, (1926)

14 A. Piccard and R. Stahel, Comptes Rendus, 185, 1198, (1927)

6 G. Joos, Ann. d. Physik, 7, 385, (1930)

5 W. M. Hicks, Phil. Mag., 3, 9, (1902)

8 H. A. Lorentz, "The Theory of Electrons", B. G. Teubner, Leipsig, (1916)

gives a discussion of the theory of the ether drift experiment, and derives the well known result: $2 \frac{l}{\lambda} \frac{v^2}{c^2}$.

The theory of the ether drift experiment was also the subject of articles by A. Righi[15,16] who argued that there should be no fringe shift detectable because while there is a difference in the lengths of the two paths, upon rotation of the interferometer, the two beams exchange places and hence cancel the effect. E. R. Hedrick[4] and Paul S. Epstein[2] both presented papers on the subject at the ether drift conference held in Pasadena, California, in 1927. These papers dealt mainly with the effect produced when the mirrors are slightly oblique to the arms.

In his discussion of the regular ninety-degree interferometer, Epstein states that the angle between the rays in their final paths is equal to: $\frac{v^2}{c^2} \cos 2\theta$. This corresponds to the angle H, given in this paper by equation 26. Comparison of this value given by Epstein and the angle given by equation 26 shows that in the case of the interferometer with arms inclined forty-five degrees, the angle between the rays is greater.

The "Ether Drift Conference"[3] mentioned above shows the amount of attention which physicists have given to the ether

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[15] A. Righi, Comptes Rendus, 168, 837, (1919)

[16] A. Righi, Comptes Rendus, 170, 497 and 1550, (1920)

[4] E. R. Hedrick, Astrophys. J., 68, 374, (1928)

[2] P. S. Epstein, Astrophys. J., 68, 383, (1928)

[3] Ether Drift Conference, Astrophys. J., 68, 352, (1928)

drift problem. On February 4 and 5, 1927, a special conference was held at the Mount Wilson Observatory in Pasadena, California, which was attended by many prominent physicists. The entire conference was devoted to the subject of measuring the ether drift, and reports on various phases of the problem were presented by Michelson, Lorentz, Miller, Kennedy,Hedrick, and Epstein. Many others joined in the informal discussion.

Finally, we come to W. B. Cartmel[1], whom the author heard last November at the meeting of the American Physical Society at Wellesley, Massachusetts. At that time, Cartmel presented, in abstract form, an article claiming that for the greatest fringe shift effect the arms of an interferometer should be inclined forty-five degrees to each other. Mr. Cartmel's formula gave the expected fringe shift as a function of the angle between the arms of the interferometer, as well as the angle of drift. However, due to the fact (as pointed out in the conclusion) that this formula apparently predicted an effect when the arms are taken parallel to each other, the writer decided to attempt his own investigation of the ether drift problem.

The writer has carefully looked over many reports, both of experimental investigations and of theoretical discussions, of the ether drift experiment, and with the exception of the statement by Cartmel, he has not found a single suggestion of the fact that the effect can be increased by using a forty-five degree angle of inclination. With the exception of the

[1] W. B. Cartmel, Physical Review, 53, 108, (1938)

interferometer of Kennedy, all others seem to have been rectangular, and Kennedy shows no trace of having used an oblique angled instrument with the expectation of increasing the effect. The writer believes, consequently, that this paper may help to call attention to a fact long overlooked.

1. EFFECT OF THE MOTION UPON THE RAY FROM THE SOURCE.

The first noticeable effect of the motion of the apparatus is the fact that the ray from the source does not strike the plate in a direction parallel to the source arm, but in a direction slightly inclined to the source arm.

In order to establish this fact, let us refer to plate 1, and assume a system of axes intersecting at the source, the x' axis parallel to the source arm, the y' axis perpendicular to it. Let P represent the position of the plate at the instant when a spark is emitted at S. The spark travels in a stationary ether, through which the entire apparatus is moving with a velocity of drift v, inclined at an angle Θ with the x' axis. By the time the spark overtakes the plate, the plate will have advanced to a new position, as shown at O. Since the spark has been traveling in an assumed stationary medium, it will appear to have come from point S, and not from some new position now occupied by the source. Hence the incident ray will make a small angle, PSO, with the source arm, which we shall designate ε. We shall now find the measure of this angle ε, and show that it is independent of SP, the distance from the source to the plate.

The distance SO is traveled by the ray with the velocity c in exactly the same length of time as is required by the plate, moving with the velocity of drift v, to travel the distance PO. Let t equal this common time.

Then: $SO = ct$ and $PO = vt$

Hence: $$\frac{ct}{vt} = \frac{\sin(\pi - \theta)}{\sin \varepsilon}$$

or $$\sin \varepsilon = \frac{v}{c} \sin \theta \qquad \text{(Eq. 1)}$$

Since in this investigation we shall include no terms beyond the second order of v/c, we may accordingly let:

$$\varepsilon = \sin \varepsilon = \frac{v}{c} \sin \theta \qquad \text{(Eq. 2)}$$

2. ANGLE OF REFLECTION OF THE RAY FROM PLATE O, ALONG THE ARM A.

It is a well known fact that a ray of light, incident upon a moving plate will, unless the motion is parallel to the plate, be reflected away from the plate in such a way that the angle of reflection is not exactly equal to the angle of incidence.

If: ϕ = the angle of incidence

ϕ' = the angle of reflection

c = the velocity of light

u = the velocity of the plate perpendicular to itself, positive when the plate moves away from the source

then the relation between the angles of incidence and reflection is given by the following equation:

$$\tan \frac{\phi'}{2} = \frac{c + u}{c - u} \tan \frac{\phi}{2} \qquad \text{(Eq. 3)} \quad (1)$$

We shall now designate as ϕ' the angle of reflection of the ray, as it is reflected from the plate, down along the arm A of the interferometer. We shall assume a set of co-ordinate

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(1) For proof of eq. 3, see the appendix A, page 73.

axes intersecting at O, the position occupied by the plate at the instant when the ray from S is incident, the x axis extending in the direction of the source arm and arm B, with the y axis perpendicular. The orientation of the apparatus with respect to the co-ordinate axes is shown on plate 2.

As is clearly shown on plate 2, where ϕ is the angle of incidence:

$$\phi = \frac{3}{8}\pi - \varepsilon \qquad \text{(Eq. 4)}$$

and u, the component of the velocity of the plate perpendicular to the plate, is given by:

$$u = v \sin(\theta + \frac{\pi}{8}) \qquad \text{(Eq. 5)}$$

Substituting now in equation 3, from equations 4 and 5:

$$\tan\frac{\phi_1'}{2} = \frac{c + v\sin(\theta + \pi/8)}{c - v\sin(\theta + \pi/8)} \tan(\frac{3}{16}\pi - \frac{\varepsilon}{2}) \qquad \text{(Eq. 6)}$$

The first factor of the right hand side of equation 6 may be expanded into a series as follows:

$$\frac{c/c + (v/c)\sin(\theta + \pi/8)}{c/c - (v/c)\sin(\theta + \pi/8)} = 1 + 2(v/c)\sin(\theta + \pi/8) + 2(v^2/c^2)\sin^2(\theta + \pi/8) + \ldots\ldots \qquad \text{(Eq. 7)}$$

Substituting $(v/c)\sin\theta$ for ε in the second factor of equation 6 and expanding the resulting quotient into a series, we have:

$$\tan(\frac{3}{16}\pi - \frac{v}{2c}\sin\theta) = \frac{\tan(3/16)\pi - \tan[(v/2c)\sin\theta]}{1 + \tan(3/16)\pi \tan[(v/2c)\sin\theta]}$$

$$= \tan(3/16)\pi - [\sec^2(3/16)\pi][(v/2c)\sin\theta] + [\tan(3/16)\pi][\sec^2(3/16)\pi][(v^2/4c^2)\sin^2\theta] \qquad \text{(Eq. 8)}$$

The product of the right hand members of equations 7 and 8 result in the following equation, which defines ϕ_1', the angle of reflection of the ray along arm A:

$$\tan\frac{\phi_1'}{2} = \tan\frac{3}{16}\pi + \frac{v}{c}\left[2\tan\frac{3}{16}\pi\sin\left(\theta+\frac{\pi}{8}\right) - \frac{1}{2}\sec^2\frac{3}{16}\pi\sin\theta\right]$$
$$+\frac{v^2}{c^2}\left[2\tan\frac{3}{16}\pi\sin^2\left(\theta+\frac{\pi}{8}\right) + \frac{1}{4}\tan\frac{3}{16}\pi\sec^2\frac{3}{16}\pi\sin^2\theta\right.$$
$$\left. - \sin\theta\sin\left(\theta+\frac{\pi}{8}\right)\sec^2\frac{3}{16}\pi\right] \qquad \text{(Eq. 9)}$$

Values of the constants appearing in equation 9, taken from an original table by Briggs, in the possession of the Boston Public Library, and correct to ten decimal places, are as follows:

$\tan(3/16)\pi = \tan 33^\circ\ 45' = .6681786378$

$\sin(\pi/8) = \sin 22.5^\circ = .3826834324$

$\cos(\pi/8) = \cos 22.5^\circ = .9238795324$

$\sin(\pi/4) = \cos(\pi/4) = .7071067813$

From these may be determined: $\sec^2(3/16)\pi = 1.4464626920$

Hence, equation 9 can be written, by substituting the numerical values of these constants, as:

$$\tan(\phi_1'/2) = .6681786378 + (v/c)[.5114017893(\sin\theta + \cos\theta)]$$
$$+(v^2/c^2)[.0459188760\sin^2\theta + .3914099850\sin\theta\cos\theta$$
$$+ .1957049920\cos^2\theta] \qquad \text{(Eq. 10)} \quad (1)$$

3. ANGLE OF REFLECTION OF THE RAY FROM THE MIRROR AT THE END OF ARM A

As may be seen from equation 10, $\tan(\phi_1'/2) > \tan(3/16)\pi$, and therefore $\phi_1' > (3/8)\pi$. Referring now to plate 3, where the origin of co-ordinates has again been taken at O, the point of

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(1) For a detailed derivation of equation 10, see appendix B, page 75.

incidence of the ray from the source upon the plate, and denote by t the length of time required for the passage of the ray from O to F, the point of incidence of the ray upon mirror A. During this time t, the mirror will advance to a new position (marked position 2) but since it remains parallel to itself, it is not necessary to take into consideration this change in position while finding the angle of reflection.

Denote by ϕ_1'' the angle of reflection at F. The velocity of mirror A in a direction perpendicular to itself, is given by: $v\cos(\theta + \frac{\pi}{4})$. Hence, substituting again in equation 3, we have:

$$\tan\frac{\phi_1''}{2} = \frac{c + v\cos[\theta + (\pi/4)]}{c - v\cos[\theta + (\pi/4)]}\tan\frac{1}{2}(\phi_1' - \frac{3}{8}\pi) \qquad \text{(Eq. 11)}$$

The first factor of the right hand side of equation 11 may be expanded into a series as follows:

$$\frac{1 + \frac{v}{c}\cos(\theta + \frac{\pi}{4})}{1 - \frac{v}{c}\cos(\theta + \frac{\pi}{4})} = 1 + \frac{2v}{c}\cos(\theta + \frac{\pi}{4}) + \frac{2v^2}{c^2}\cos^2(\theta + \frac{\pi}{4}) + \cdots\cdots$$

$$= 1 + \frac{2v}{c}[.7071067813(\cos\theta - \sin\theta)] + 2\frac{v^2}{c^2}[\frac{1}{2}(\cos\theta - \sin\theta)^2]$$

$$= 1 + \frac{v}{c}[1.4142135626(\cos\theta - \sin\theta)] + \frac{v^2}{c^2}[1 - 2\sin\theta\cos\theta] \qquad \text{(Eq. 12)}$$

The second factor may be expanded into a series in the following manner:

$$\tan(\frac{\phi_1'}{2} - \frac{3\pi}{16}) = \frac{\tan\frac{\phi_1'}{2} - \tan\frac{3\pi}{16}}{1 + \tan\frac{3\pi}{16}\tan\frac{\phi_1'}{2}}$$

Using the result of equation 10, we may write that the numerator of the above is:

$$\frac{v}{c}[.5114017893(\sin\theta + \cos\theta)] + \frac{v^2}{c^2}[.0459188760 \sin^2\theta + .3914099850 \sin\theta\cos\theta + .1957049920 \cos^2\theta]$$

and the denominator is:

$$1.4464626920 + \frac{v}{c}[.3417077509 (\sin\theta + \cos\theta)] + \frac{v^2}{c^2}[.0306820120 \sin^2\theta + .2615317906 \sin\theta\cos\theta + .1307658950 \cos^2\theta]$$

This denominator may be divided into the numerator to give a quotient which is as follows:

$$\tan\left(\frac{\phi_1'}{2} - \frac{3\pi}{16}\right) = \frac{v}{c}[.3535533907(\sin\theta + \cos\theta)] + \frac{v^2}{c^2}[-.0517766952 \sin^2\theta + .1035533912 \sin\theta\cos\theta + .0517766953 \cos^2\theta] \qquad \text{(Eq. 13)}$$

Equation 11 shows tha t the product of equations 12 and 13 gives the value of $\tan\frac{\phi_1''}{2}$, or:

$$\tan\frac{\phi_1''}{2} = \frac{v}{c}[.3535533907(\sin\theta + \cos\theta)] + \frac{v^2}{c^2}[-.5517766954 \sin^2\theta + .103553391 \sin\theta\cos\theta + .5517766954 \cos^2\theta] \qquad \text{(Eq. 14)}$$

Since, to second order approximation, $\tan\phi_1'' = \phi_1''$,, we have:

$$\phi_1'' = .7071067814 (\sin\theta + \cos\theta)\frac{v}{c} + \frac{v^2}{c^2}[1.1035533908 \cos^2\theta + .2071067816 \sin\theta\cos\theta - 1.1035533908 \sin^2\theta] \qquad \text{(Eq. 15)}$$

4. ANGLE OF REFLECTION OF THE RAY AT MIRROR B

We shall determine now the angle of reflection of the ray at mirror B. Referring to plate 4, it is evident that the angle of incidence at mirror B equals angle ε, which was defined

by equation 2. The velocity u, of mirror B, perpendicular to itself, is: $u = v \cos \Theta$. Hence, if we denote by ϕ'_2 the angle of reflection, we have:

$$\tan \frac{\phi'_2}{2} = \frac{c + v \cos \Theta}{c - v \cos \Theta} \tan \frac{1}{2} \left(\frac{v}{c} \sin \Theta\right)$$

$$= \frac{1 + \frac{v}{c} \cos \Theta}{1 - \frac{v}{c} \cos \Theta} \tan \frac{1}{2} \left(\frac{v}{c} \sin \Theta\right) \qquad \text{(Eq. 16)}$$

Expanding the fraction into a series and using the fact that to second order approximations, $\tan \frac{1}{2} \left(\frac{v}{c} \sin \Theta\right) = \frac{1}{2}\left(\frac{v}{c} \sin \Theta\right)$ we have:

$$\tan \frac{\phi'_2}{2} = \frac{v}{2c} \sin \Theta + \frac{v^2}{c^2} \sin \Theta \cos \Theta + \cdots\cdots\cdots \qquad \text{(Eq. 17)}$$

5. ANGLE OF REFLECTION OF THE RAY FROM THE PLATE, AFTER RETURNING FROM MIRROR B

We will refer once more to plate 4, which shows that during the passage of the ray from O to B and back again to the plate at T, the plate has advanced in position, due to the velocity of drift, but has, however, remained parallel to itself, and consequently in figuring the angle of reflection from the plate at T, this change of position may be neglected. Let ϕ''_2 denote this angle of reflection. It is evident from plate 4 that the angle of incidence with the plate at T is $\left(\frac{3}{8}\pi + \phi'_2\right)$ and that the component of the velocity of drift in the direction perpendicular to the plate is: $v \sin\left(\Theta + \frac{\pi}{8}\right)$. Therefore, substituting in

equation 3, we have the angle ϕ''_2 defined by:

$$\tan\frac{\phi''_2}{2} = \frac{c - v\sin(\theta + \frac{\pi}{8})}{c + v\sin(\theta + \frac{\pi}{8})} \tan\frac{1}{2}(\frac{3}{8}\pi + \phi'_2) \qquad \text{(Eq. 18)}$$

In a manner similar to that used in the case of arm A, the first factor of the above equation (the fraction) may be expanded into the series:

$$\frac{1 - \frac{v}{c}\sin(\theta + \frac{\pi}{8})}{1 + \frac{v}{c}\sin(\theta + \frac{\pi}{8})} = 1 - \frac{2v}{c}\sin(\theta + \frac{\pi}{8}) + \frac{2v^2}{c^2}\sin^2(\theta + \frac{\pi}{8}) + \cdots \qquad \text{(Eq. 19)}$$

The second factor may be expanded thus:

$$\tan(\frac{3\pi}{16} + \frac{\phi'_2}{2}) = \frac{\tan\frac{3\pi}{16} + \tan\frac{\phi'_2}{2}}{1 - \tan\frac{3\pi}{16}\tan\frac{\phi'_2}{2}}$$

Expanding this quotient into a series and evaluating the terms of the series, gives:

$$\tan(\frac{3\pi}{16} + \frac{\phi'_2}{2}) = .6681786378 + .7232313460 \sin\theta\frac{v}{c} + \frac{v^2}{c^2}[.2416238677 \sin^2\theta + 1.4464626920 \sin\theta\cos\theta] \qquad \text{(Eq. 20)}$$

Evaluating the right hand member of equation 19, we obtain

$$1 - (1.8477590648 \sin\theta + .7653668648 \cos\theta)\frac{v}{c} + \frac{v^2}{c^2}(1.7071067808 \sin^2\theta + 1.4142135626 \sin\theta\cos\theta + .2928932188 \cos^2\theta)$$

and the product of this by the right hand side of equation 20

results in the following equation:

$$\tan\frac{\phi''_2}{2} = .6681786378 - .5114017891\,(\sin\theta + \cos\theta)\frac{v}{c} + \frac{v^2}{c^2}[.0459188757\sin^2\theta + 1.8378726763\sin\theta\cos\theta + .1957049920\cos^2\theta] \qquad \text{(Eq. 21)}$$

6. FINAL ANGLE BETWEEN THE RAYS

Before continuing the investigation further, it will be well to determine the final angle between the two rays, one as it is reflected from mirror A and passes through the plate, the other as it returns from mirror B and is reflected from the plate. Plate 5 shows clearly the paths of the two rays. It is important to determine this angle between the rays, since if the rays should happen to be parallel, no fringes will be formed.

Referring to the sketch on the previous page, it is seen that the angle between the rays is:

$$\text{Angle between rays} = \frac{5}{8}\pi - \phi''_2 - \frac{\pi}{4} - \phi''_1 = \frac{3}{8}\pi - \phi''_1 - \phi''_2 \qquad \text{(Eq. 22)}$$

The equations defining ϕ''_1 and ϕ''_2 as already found, are:

$$\frac{\phi''_1}{2} = \frac{v}{c}[.3535533907\,(\sin\theta + \cos\theta)] + \frac{v^2}{c^2}\left[\frac{\cos^2\theta - \sin^2\theta}{2} + .0517766954\,(\cos^2\theta + 2\sin\theta\cos\theta - \sin^2\theta)\right] \qquad \text{(Eq. 15)}$$

$$\frac{\phi''_2}{2} = .6681786378 - .5114017891\,(\sin\theta + \cos\theta)\frac{v}{c} + \frac{v^2}{c^2}$$
$$[.0459188757\sin^2\theta + 1.8378726763\sin\theta\cos\theta$$
$$+ .1957049920\cos^2\theta] \qquad \text{(Eq. 21)}$$

From equation 21, it is evident that ϕ''_2 differs from $\frac{3}{8}\pi$ by a small angle. Call this small angle δ and proceed to find δ as follows:

$$\text{Let } \phi''_2 = \frac{3}{8}\pi + \delta \qquad (\delta \text{ is a small angle}) \qquad \text{(Eq. 23)}$$

then: $$\frac{\phi''_2}{2} = \frac{3\pi}{16} + \frac{\delta}{2}$$

Also, let $\tan\frac{\phi''_2}{2} - \tan\frac{3}{16}\pi = \Sigma$ (where $\Sigma = f(\frac{v}{c}, \frac{v^2}{c^2}, \text{etc.})$

Then: $$\tan\frac{\phi''_2}{2} = \tan\left(\frac{3\pi}{16} + \frac{\delta}{2}\right) = \frac{\tan\frac{3}{16}\pi + \tan\frac{\delta}{2}}{1 - \tan\frac{3\pi}{16}\tan\frac{\delta}{2}} = \tan\frac{3\pi}{16} + \Sigma$$

Clearing of fractions:

$$\tan\frac{3\pi}{16} + \tan\frac{\delta}{2} = \tan\frac{3\pi}{16} + \Sigma - \tan^2\frac{3\pi}{16}\tan\frac{\delta}{2} - \Sigma\tan\frac{3\pi}{16}\tan\frac{\delta}{2}$$

Reducing, this becomes:

$$\tan\frac{\delta}{2} + \tan^2\frac{3\pi}{16}\tan\frac{\delta}{2} + \Sigma\tan\frac{3\pi}{16}\tan\frac{\delta}{2} = \Sigma$$

$$\tan\frac{\delta}{2}\left(1 + \tan^2\frac{3\pi}{16} + \Sigma\tan\frac{3\pi}{16}\right) = \tan\frac{\delta}{2}\left(\sec^2\frac{3\pi}{16} + \tan\frac{3\pi}{16}\Sigma\right) = \Sigma$$

Hence: $\tan \frac{\delta}{2} = \frac{\Sigma}{\sec^2 \frac{3\pi}{16} + \tan \frac{3\pi}{16} \Sigma} = \frac{\Sigma}{\tan \frac{3\pi}{16} \left(\frac{\sec^2 \frac{3\pi}{16}}{\tan \frac{3\pi}{16}} + \Sigma \right)}$

Referring to equation 21, we see that

$$\Sigma = -.5114017891 \, (\sin \theta + \cos \theta) \frac{v}{c} + \frac{v^2}{c^2} [.0459188757 \sin^2 \theta + 1.8378726763 \sin \theta \cos \theta + .1957049920 \cos^2 \theta]$$

set $\frac{\sec^2 \frac{3\pi}{16}}{\tan \frac{3\pi}{16}} = K$ and let $\Sigma = A \frac{v}{c} + B \frac{v^2}{c^2}$

then $\tan \frac{\delta}{2} = \frac{A \frac{v}{c} + B \frac{v^2}{c^2}}{\tan \frac{3\pi}{16} \left[K + A \frac{v}{c} + B \frac{v^2}{c^2} \right]} = \frac{A}{K} \frac{v}{c} + \left(\frac{B}{K} + \frac{A^2}{K^2} \right) \frac{v^2}{c^2}$

Therefore: $\tan \frac{\delta}{2} = \frac{1}{\tan \frac{3\pi}{16}} \left[\frac{\tan \frac{3\pi}{16} A}{\sec^2 \frac{3\pi}{16}} \frac{v}{c} + \left(\frac{\tan \frac{3\pi}{16} B}{\sec^2 \frac{3\pi}{16}} - \frac{\tan^2 \frac{3\pi}{16} A^2}{\sec^4 \frac{3\pi}{16}} \right) \frac{v^2}{c^2} \right]$

$$= \frac{A \frac{v}{c} + B \frac{v^2}{c^2}}{\sec^2 \frac{3\pi}{16}} - \frac{A^2 \tan \frac{3\pi}{16}}{\sec^4 \frac{3\pi}{16}} \frac{v^2}{c^2}$$

Since $\Sigma = A \frac{v}{c} + B \frac{v^2}{c^2}$, $\Sigma^2 = A^2 \frac{v^2}{c^2}$ (to second order terms)

and we may write:

$$\tan \frac{\delta}{2} = \frac{\Sigma}{\sec^2 \frac{3\pi}{16}} - \frac{\Sigma^2 \tan \frac{3\pi}{16}}{\sec^4 \frac{3\pi}{16}} \qquad \text{(Eq. 24)}$$

Evaluating equation 22 in terms of known constants, and replacing Σ in terms of $\frac{v}{c}$, $\frac{v^2}{c^2}$, we get:

$$\tan \frac{\delta}{2} = -.3535533905(\sin \theta + \cos \theta)\frac{v}{c} + \frac{v^2}{c^2}[-.05177669545 \sin^2 \theta + 1.103553390 \sin \theta \cos \theta + .0517766952 \cos^2 \theta]$$

from which we derive the following:

$$\delta = -.7071067810(\sin \theta + \cos \theta)\frac{v}{c} + \frac{v^2}{c^2}[-.10355339090 \sin^2 \theta + 2.207106780 \sin \theta \cos \theta + .1035533904 \cos^2 \theta] \qquad \text{(Eq. 25)}$$

As was stated in equation 22, the angle between the rays is: $\frac{3}{8}\pi - \phi''_2 - \phi''_1$

Also, by equation 23, $\phi''_2 = \frac{3}{8}\pi + \delta$

Consequently, the angle between the rays is:

$$\frac{3}{8}\pi - \frac{3}{8}\pi - \delta - \phi''_1 = -\delta - \phi''_1$$

This may be evaluated by means of equations 25 and 15. If we designate as H the angle between the rays, the result is:

$$H = \frac{v^2}{c^2}[1.2071067817 \sin^2 \theta - 1.2071067812 \cos^2 \theta - 2.4142135616 \sin \theta \cos \theta] \qquad \text{(Eq. 26)}$$

6(a). THE ANGLE BETWEEN THE ARM A AND THE RAY FROM O, GOING TOWARD MIRROR A

An inspection of equation 9 shows that ϕ'_1 exceeds $\frac{3}{8}\pi$ by a small angle, and this excess is exactly the angle between the

arm A and the ray from O along the arm. Since we shall soon want to know the magnitude of this angle we shall proceed to find it as follows:

Let $\phi_1' = \frac{3}{8}\pi + \delta'$ or $\frac{\phi_1'}{2} = \frac{3}{16} + \frac{\delta'}{2}$

Then: $$\tan\frac{\phi_1'}{2} = \tan\left(\frac{3\pi}{16} + \frac{\delta}{2}\right) = \frac{\tan\frac{3\pi}{16} + \tan\frac{\delta'}{2}}{1 - \tan\frac{3\pi}{16}\tan\frac{\delta'}{2}}$$

Also let: $$\tan\frac{\phi_1'}{2} = \tan\frac{3\pi}{16} + \Sigma'$$

Reference to equation 10 shows that the value of Σ' is:

$$\Sigma' = .5114017893(\sin\theta + \cos\theta)\frac{v}{c} + \frac{v^2}{c^2}(.0459188760\sin^2\theta + .3914099850\sin\theta\cos\theta + .1957049920\cos^2\theta)$$

Consequently: $$\frac{\tan\frac{3\pi}{16} + \tan\frac{\delta'}{2}}{1 - \tan\frac{3\pi}{16}\tan\frac{\delta'}{2}} = \tan\frac{3\pi}{16} + \Sigma'$$

Clearing of fractions:

$$\tan\frac{3\pi}{16} + \tan\frac{\delta'}{2} = \tan\frac{3\pi}{16} + \Sigma' - \tan^2\frac{3\pi}{16}\tan\frac{\delta'}{2} - \Sigma'\tan\frac{3\pi}{16}\tan\frac{\delta'}{2}$$

Solving this for $\frac{\delta'}{2}$ we have:

$$\tan\frac{\delta'}{2} = \frac{\Sigma'}{\sec^2\frac{3\pi}{16} + \tan\frac{3\pi}{16}\Sigma'} = \frac{\Sigma'}{\left(\sec^2\frac{3\pi}{16}\right)\tan\frac{3\pi}{16}\left(\frac{}{\left(\tan\frac{3\pi}{16}\right)} + \Sigma'\right)}$$

Again set $\frac{\sec^2 \frac{3\pi}{16}}{\tan \frac{3\pi}{16}} = K$ and let $\Sigma' = A\frac{v}{c} + B\frac{v^2}{c^2}$

$$\text{Then: } \tan \frac{\delta'}{2} = \frac{A\frac{v}{c} + B\frac{v^2}{c^2}}{\tan \frac{3\pi}{16}(K + A\frac{v}{c} + B\frac{v^2}{c^2})} = \frac{1}{\tan \frac{3\pi}{16}}\left[\frac{A}{K}\frac{v}{c} + \left(\frac{B}{K} - \frac{A^2}{K^2}\right)\frac{v^2}{c^2}\right]$$

Replacing the value of K:

$$\tan \frac{\delta'}{2} = \frac{1}{\tan \frac{3\pi}{16}}\left[\frac{\tan \frac{3\pi}{16} A}{\sec^2 \frac{3\pi}{16}}\frac{v}{c} + \left(\frac{\tan \frac{3\pi}{16} B}{\sec^2 \frac{3\pi}{16}} - \frac{\tan^2 \frac{3\pi}{16} A^2}{\sec^4 \frac{3\pi}{16}}\right)\frac{v^2}{c^2}\right]$$

$$= \frac{A}{\sec^2 \frac{3\pi}{16}}\frac{v}{c} + \left[\frac{B}{\sec^2 \frac{3\pi}{16}} - \frac{A^2 \tan \frac{3\pi}{16}}{\sec^4 \frac{3\pi}{16}}\right]\frac{v^2}{c^2}$$

Evaluating this in terms of known constants, results in:

$$\tan \frac{\delta'}{2} = .3535533907(\sin \theta + \cos \theta)\frac{v}{c} + \frac{v^2}{c^2}(.05177669518 \cos^2 \theta + .1035533913 \sin \theta \cos \theta - .05177669528 \sin^2 \theta)$$

(Eq. 27)

7. LENGTH OF PATH FROM PLATE TO THE MIRROR A

We refer now to plate 6. The angle between the direction of drift and the direction taken by the ray along arm A is $(\frac{\pi}{4} + \theta - \delta')$. The components of v in the directions along and parallel to the ray are:

parallel component = $v \cos(\theta + \frac{\pi}{4} - \delta')$

perpendicular component = $v \sin(\theta + \frac{\pi}{4} - \delta')$

while the ray is following the pa th OG, the mirror moves through a distance vt. Let NG represent the perpendicular mirror, and consider the motion of point G, on the perpendicular mirror, while the ray pa sses from O to its intersection with the new position of the mirror. Point G, which will also move through the distance vt, may be thought of as having two motions. The component of vt a long the ra y carries G to a new position F', but the component perpendicular to the ray carries F' to a new point F". Since we are interested in the pa th from O, making the small a ngle δ' with OM, to the intersection of the ra y with the mirror passing through F", the actual path of the ray is OF.

We refer aga in to plate 6, and consider the triangle FF'F"

angle FF"F' = δ'
angle F"F'F = 90°

Let t = time required for the ray to go from O to F. Then:

$$\tan \delta' = \frac{F F'}{v t \sin(\theta + \frac{\pi}{4} - \delta')}$$

Therefore: $F F' = vt \sin(\theta + \frac{\pi}{4} - \delta') \tan \delta'$

Then, since: OF = OG + GF' - FF', we have:

$$OF = L \sec \delta' + vt \cos(\theta + \frac{\pi}{4} - \delta') - vt \sin(\theta + \frac{\pi}{4} - \delta') \tan\delta'$$

Also, OF = ct, so:

$$ct = L \sec \delta' + vt \cos(\theta + \frac{\pi}{4} - \delta') - vt \sin(\theta + \frac{\pi}{4} - \delta') \tan \delta'$$

Solving for t:

$$t = \frac{L \sec \delta'}{c + v[\tan \delta' \sin(\theta + \frac{\pi}{4} - \delta') - \cos(\theta + \frac{\pi}{4} - \delta')]}$$

Since δ' is a small angle, sec δ' may be expanded in series, and tan δ' replaced by δ', so we get:

$$t = \frac{L(1 + \frac{\delta'^2}{2})}{c + v[\delta' \sin(\theta + \frac{\pi}{4} - \delta') - \cos(\theta + \frac{\pi}{4} - \delta')]}$$

and since OF = ct we have:

$$OF = \frac{L(1 + \frac{\delta'^2}{2})}{1 + \frac{v}{c}[\delta' \sin(\theta + \frac{\pi}{4} - \delta') - \cos(\theta + \frac{\pi}{4} - \delta')]} \qquad \text{(Eq. 28)}$$

Treating the coefficient of $\frac{v}{c}$ in the denominator,

$$\delta' \sin[(\theta + \frac{\pi}{4}) - \delta'] = \frac{\delta'}{\sqrt{2}}[\sin(\theta - \delta') + \cos(\theta - \delta')]$$

expanding on the right, replacing sin δ' by δ' and cos δ' by $(1 - \frac{1}{2}\delta'^2)$ we obtain for the right hand mamber:

$$= \frac{1}{\sqrt{2}}[\delta' \sin\theta - \frac{\delta'^3}{2}\sin\theta - \delta'^2 \cos\theta + \delta' \cos\theta - \frac{\delta'^3}{2}\cos\theta + \delta'^2 \sin\theta]$$

Since the above is to be multiplied by $\frac{v}{c}$, and since δ' is itself a function of $\frac{v}{c}$, $\frac{v^2}{c^2}$, etc., all the terms within the bracket involving more than the first power of δ' will vanish from the product, so that the only remaining terms will be:

$$\frac{v}{c}\frac{1}{\sqrt{2}}[\delta' \sin\theta + \delta' \cos\theta] = \frac{\delta'}{\sqrt{2}}\frac{v}{c}(\sin\theta + \cos\theta)$$

In the same manner, treating the term:

$$\cos[(\theta + \frac{\pi}{4}) - \delta'] = \frac{1}{\sqrt{2}}[\cos\theta\cos\delta' + \sin\theta\sin\delta' - \sin\theta\cos\delta' + \cos\theta\sin\delta']$$

$$= \frac{1}{\sqrt{2}}[\cos\theta - \frac{\delta'^2}{2}\cos\theta + \delta'\sin\theta - \sin\theta + \frac{\delta'^2}{2}\sin\theta + \delta'\cos\theta]$$

Multiplying by $\frac{v}{c}$, and dropping out those terms which, as a result, would be of higher order than the second in $\frac{v}{c}$, we have:

$$= \frac{1}{\sqrt{2}}\frac{v}{c}[(\cos\theta - \sin\theta) + \delta'(\cos\theta + \sin\theta)]$$

Hence, the denominator of OF is:

$$1 + \frac{v}{c}[\frac{1}{\sqrt{2}}\delta'(\sin\theta + \cos\theta) - \frac{1}{\sqrt{2}}(\cos\theta - \sin\theta) - \frac{1}{\sqrt{2}}\delta'(\sin\theta + \cos\theta)]$$

$$= 1 - \frac{1}{\sqrt{2}}\frac{v}{c}(\cos\theta - \sin\theta)$$

Referring to equation 27, we see that δ' is a function of $\frac{v}{c}$ and $\frac{v^2}{c^2}$, and consequently δ'^2 will involve only the square of the first order term. In fact:

$$\frac{\delta'^2}{2} = \frac{1}{4}(\sin\theta + \cos\theta)^2\frac{v^2}{c^2}$$

From this point on in the development of the problem it will often be found convenient to make use of the following substitutions:

let: $\cos\theta + \sin\theta = A$

$\cos\theta - \sin\theta = B$ (Equations 29)

Employing these substitutions and making use of the reduction of the denominator of equation 28 as already shown, we may write:

$$OF = \frac{L(1 + \frac{A^2}{4}\frac{v^2}{c^2})}{1 - \frac{B}{\sqrt{2}}\frac{v}{c}}$$

and expanding this into a series and keeping only up to second order terms, we have:

$$OF = L[1 + \frac{B}{\sqrt{2}}\frac{v}{c} + \frac{A^2 + 2B^2}{4}\frac{v^2}{c^2}] \qquad \text{(Eq. 30)}$$

We refer again to plate 6 and let FD be drawn from F parallel to the arm A. In order to determine the length of path from F back to the plate, it will be necessa ry to find the length FD. In the triangle FOD we have:

$$\angle FDO = \frac{3}{4}\pi - \theta$$

$$\angle OFD = \delta'$$

$$\angle FOD = \theta + \frac{\pi}{4} - \delta'$$

$$\frac{FD}{FO} = \frac{\sin(\theta + \frac{\pi}{4} - \delta')}{\sin(\frac{3}{4}\pi - \theta)} \qquad \text{Hence: } FD = \frac{\sin[(\theta + \frac{\pi}{4}) - \delta']}{\sin(\theta + \frac{\pi}{4})} OF$$

$$= \left[\cos\delta' - \sin\delta' \frac{\frac{1}{\sqrt{2}}(\cos\theta - \sin\theta)}{\frac{1}{\sqrt{2}}(\sin\theta + \cos\theta)}\right] OF$$

$$= (\cos\delta' - \frac{B}{A}\sin\delta')\, OF$$

Referring to equation 30 for the value of OF, and making approximations for sin δ' and cos δ' good to second order

terms, we have:

$$FD = (1 + \frac{B}{\sqrt{2}}\frac{v}{c} + \frac{A^2 + 2B^2}{4}\frac{v^2}{c^2})(1 - \frac{B}{A}\delta' - \frac{1}{2}\delta'^2)\,L \qquad \text{(Eq.31)}$$

Since δ' (see equation 27) is a function of $\frac{v}{c}$, $\frac{v^2}{c^2}$, etc., certain terms may be omitted from the product. Hence:

$$FD = [1 - \frac{B}{A}\delta' - \frac{1}{2}\delta'^2 + \frac{B}{\sqrt{2}}\frac{v}{c} - \frac{B^2}{\sqrt{2}A}\frac{v}{c}\delta' + \frac{A^2 + 2B^2}{4}\frac{v^2}{c^2}]\,L$$

(Eq. 32)

By referring to equation 27, it can be seen that if we make the substitution:

$$\text{let } K = .0517766954 \qquad \text{(Eq. 33)}$$

and define δ' by the following:

$$\delta' = \frac{A}{\sqrt{2}}\frac{v}{c} + \frac{v^2}{c^2}[\,2K(\cos^2\theta + 2\sin\theta\cos\theta - \sin^2\theta)] \qquad \text{(Eq. 34)}$$

we have not altered the first degree term and have still preserved nine place accuracy in the term of second degree. From this point on we shall make frequent use of this simplification. Then, making use of equations 29, we may express δ' as follows:

$$\delta' = \frac{A}{\sqrt{2}}\frac{v}{c} + K(A^2 + 2AB - B^2)\frac{v^2}{c^2} \qquad \text{(Eq.35)}$$

Substituting this va lue for δ' in equation 32, we obtain:

$$FD = [\,1 - \frac{v^2}{c^2}\frac{KB}{A}(A^2 + 2AB - B^2)]\,L \qquad \text{(Eq.36)}$$

8. LENGTH OF PATH FROM MIRROR A BACK TO PLATE

We shall now refer to plate 7, showing the path from point F on the mirror A, to its intersection with the plate at a point which we sha ll designate as H. In triangle FDE:

$$\angle FDE = \frac{\pi}{4} + \theta$$

$$\angle DFE = \phi''_1$$

$$\angle FED = \frac{3\pi}{4} - \theta - \phi''_1$$

$$\frac{FD}{FE} = \frac{\sin(\frac{3\pi}{4} - \theta - \phi''_1)}{\sin(\frac{\pi}{4} + \theta)} = \frac{\sin(45^\circ + \theta + \phi''_1)}{\sin(45^\circ + \theta)}$$

Therefore:

$$FE = \frac{\frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)}{\sin(45^\circ + \theta)\cos\phi''_1 + \cos(45^\circ + \theta)\sin\phi''_1} FD$$

$$= \frac{A}{A\cos\phi''_1 + B\sin\phi''_1} FD \quad (1) \qquad \text{(Eq. 37)}$$

$$\frac{DE}{FD} = \frac{\sin\phi''_1}{\sin(135^\circ - \theta - \phi''_1)} \quad \frac{\sin\phi''_1}{\sin(45^\circ + \theta + \phi''_1)}$$

From which:

$$DE = \frac{\sqrt{2}\sin\phi''_1}{A\cos\phi''_1 + B\sin\phi''_1} FD \qquad \text{(Eq. 38)}$$

In plate 7, Q represents the position of the plate at the

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[1] From this point on, whenever simplification results from the substitutions: $A = \cos\theta + \sin\theta$ and $B = \cos\theta - \sin\theta$, these substitutions will be made without comment.

instant that the ray is reflected from F, and QP represents the position of the arm a t this instant. Point M is the intersection of DF with the direction of drift through P. O' represents the position of the plate at the instant when the ray from F is incident upon it. In the triangle FPM:

$$\angle MFP = 90°$$

$$\angle FMP = 45° + \theta$$

$$\angle FPM = 45° - \theta$$

$$FM = L - FD \qquad \text{and} \quad PM = DQ$$

$$\sin FPM = \frac{FM}{PM} \quad \text{or} \quad \sin(45° - \theta) = \frac{L - FD}{DQ}$$

$$DQ = \frac{L - FD}{\sin(45° - \theta)} = \frac{\sqrt{2}(L - FD)}{B} \qquad \text{(Eq. 39)}$$

where B as usual, equals $(\cos\theta - \sin\theta)$.

By use of equations 38 and 39 we have:

$$QE = DE + QD = \frac{\sqrt{2}\sin\phi_1''}{A\cos\phi_1'' + B\sin\phi_1''} FD + \frac{\sqrt{2}(L - FD)}{B}$$

First putting this over a common denominator and then separating the terms, this may be expressed as:

$$QE = \sqrt{2}\left[\frac{L}{B} - \frac{A\cos\phi_1'' FD}{B(A\cos\phi_1'' + B\sin\phi_1'')}\right] \qquad \text{(Eq. 40)}$$

In triangle HEO' we have:

$$\angle EO'H = \left(\frac{\pi}{8} + \theta\right)$$

$$\angle EHO' = \left(\frac{\pi}{8} + \phi_1''\right)$$

$$\angle O'EH = 135° - \theta - \phi_1'' = \angle FED$$

$$\frac{EH}{EO'} = \frac{\sin(\frac{\pi}{8} + \theta)}{\sin(\frac{\pi}{8} + \phi_1'')} = \frac{\sin\frac{\pi}{8}\cos\theta + \cos\frac{\pi}{8}\sin\theta}{\sin\frac{\pi}{8}\cos\phi_1'' + \cos\frac{\pi}{8}\sin\phi_1''} \qquad \text{(Eq. 41)}$$

Now, $\sin\frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$ and $\cos\frac{\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}$

and by substitution of these values, and subsequent simplification, equation 41 may be written:

$$\frac{EH}{EO'} = \frac{\cos\theta + (\sqrt{2} + 1)\sin\theta}{\cos\phi_1'' + (\sqrt{2} + 1)\sin\phi_1''}$$

and substituting from equation 29 we have:

$$EO' = \frac{\cos\phi_1'' + (\sqrt{2} + 1)\sin\phi_1''}{A + \frac{A - B}{\sqrt{2}}}\, EH \qquad \text{(Eq. 42)}$$

In plate 7, O' represents the position of the plate at the instant when the ray is incident upon the plate at H. Consequently:

$$\frac{QO'}{FH} = \frac{v}{c} \quad \text{or} \quad QO = \frac{v}{c}\, FH$$

$$QE + EO' = \frac{v}{c}(FE + EH)$$

Substituting in this equation values already found for QE (equation 40), EO' (equation 42) and FE (equation 37) we have:

$$\sqrt{2}\left[\frac{L}{B} - \frac{A\cos\phi_1''\, FD}{B(A\cos\phi_1'' + B\sin\phi_1'')}\right] + \frac{\sqrt{2}\cos\phi_1'' + (2 + \sqrt{2})\sin\phi_1''}{\sqrt{2}\,A + A - B}\, EH$$

$$= \frac{v}{c}\left[\frac{A \cdot FD}{A\cos\phi_1'' + B\sin\phi_1''} + EH\right] \qquad \text{(Eq. 43)}$$

Solving this equation for EH, we obtain:

$$EH = \frac{\dfrac{A(\sqrt{2}\cos\phi_1'' + \frac{v}{c}B)}{B(A\cos\phi_1'' + B\sin\phi_1'')}FD - \dfrac{\sqrt{2}\,L}{B}}{\dfrac{\sqrt{2}\cos\phi_1'' + (2+\sqrt{2})\sin\phi_1''}{(1+\sqrt{2})A - B} - \frac{v}{c}}$$

The reduction of this expression to a simple fraction results in the somewhat complicated expression for EH:

$$EH = \frac{A[(1+\sqrt{2})A - B](\sqrt{2}\cos\phi_1'' + \frac{v}{c}B)FD - L(A\cos\phi_1'' + B\sin\phi_1'')}{B\sqrt{2}(A\cos\phi_1'' + B\sin\phi_1'')[\cos\phi_1'' + (1+\sqrt{2})\sin\phi_1''] - \dfrac{[(1+\sqrt{2})A - B]\sqrt{2}}{B(A\cos\phi_1'' + B\sin\phi_1'')[(1+\sqrt{2})A - B]\frac{v}{c}}} \qquad \text{(Eq. 44)}$$

By making use of the substitutions for A, B and K as outlined in equations 29 and 33, equation 15 may be written as follows:

$$\sin\phi_1'' = \tan\phi_1'' = \phi_1'' = \frac{A}{\sqrt{2}}\frac{v}{c} \quad \frac{v^2}{c^2}[AB - KB^2 + 2KAB + KA^2] \qquad \text{(Eq. 45)}$$

$$\text{and } \cos\phi_1'' = 1 - \frac{1}{2}(\phi_1'')^2 = 1 - \frac{A^2}{4}\frac{v^2}{c^2} \qquad \text{(Eq. 46)}$$

In equation 44 it will be found useful to employ the additional substitution:

$$\text{let } M = A^2 + 2AB - B^2 \qquad \text{(Eq. 47)}$$

If this is done, we shall be a ble to write the values of the angles ϕ_1'' and δ' (see equation 27) as:

$$\phi_1'' = \frac{A}{\sqrt{2}}\frac{v}{c} \quad \frac{v^2}{c^2}[AB + KM] \qquad \text{(Eq. 48)}$$

$$\delta' = \frac{A}{\sqrt{2}} \frac{v}{c} \quad \frac{v^2}{c^2} KM$$

We shall now simplify the value given in equation 44 for EH. We shall consider that the numerator consists of two terms, the first term being the coefficient of FD. The value of FD itself may be taken from equation 36, while $\cos \phi''_1$ is defined in equation 46, so that the first term of the numerator becomes:

$$L\Big((\sqrt{2} + 2)A^2 - \sqrt{2}\, AB + \frac{v}{c}[(1 + \sqrt{2})A^2B - AB^2] + \frac{v^2}{c^2}\Big[\frac{A^3B\sqrt{2}}{4} + \sqrt{2}KB^2M - \frac{(\sqrt{2} + 2)A^4}{4} - (2 + \sqrt{2})ABKM\Big]\Big)$$

Considering now the second term of the numerator, we can show that:

$$A \cos \phi''_1 + B \sin \phi''_1 = A + \frac{v}{c}\frac{AB}{\sqrt{2}} + \frac{v^2}{c^2}[AB^2 + BKM - \frac{A^3}{4}] \quad \text{(Eq. 49)}$$

The second term of the numerator in its entirety may be shown to be equal to:

$$L\Big((\sqrt{2} + 2)A^2 - \sqrt{2}\, AB + \frac{v}{c}[(1 + \sqrt{2})A^2B - AB^2] + \frac{v^2}{c^2}\Big[-\frac{(\sqrt{2} + 2)A^4}{4} + \frac{A^3B\sqrt{2}}{4} + (\sqrt{2} + 2)ABKM + (\sqrt{2} + 2)A^2B^2 - AB^3\sqrt{2} - \sqrt{2}B^2KM\Big]\Big)$$

Since the numerator of the fraction in equation 44 is the difference of these two terms, we see that the numerator reduces to the expression:

$$L[2\sqrt{2}\, KB^2M - 2(2 + \sqrt{2})ABKM + AB^3\sqrt{2} - (2 + \sqrt{2})A^2B^2]\frac{v^2}{c^2}$$

Let us consider that the denominator of EH in equation 44 also consists of two terms, and proceed to simplify each term. Substitution of the values of sin ϕ_1'' and cos ϕ_1'' from equations 46 and 48, and subsequent reduction, will give for the first term of the denominator the result:

$$AB\sqrt{2} + \frac{v}{c}[(1 + \sqrt{2})A^2B + AB^2] + \frac{v^2}{c^2}[\frac{3 + 3\sqrt{2}}{\sqrt{2}} A^2B^2 + (\sqrt{2} + 2)ABKM$$

$$+ B^2KM\sqrt{2} + AB^3\sqrt{2} - \frac{A^3B}{\sqrt{2}}]$$

The second term of the denominator becomes:

$$\frac{v}{c}[(1 + \sqrt{2})A^2B - AB^2] + \frac{v^2}{c^2}[\frac{1 + \sqrt{2}}{\sqrt{2}} A^2B^2 - \frac{AB^3}{\sqrt{2}}]$$

The difference of these two terms gives for the denominator of EH the value:

$$AB\sqrt{2} + 2AB^2\frac{v}{c} + \frac{v^2}{c^2}[(2 + \sqrt{2})(A^2B^2 + ABKM) + B^2KM\sqrt{2} + \frac{3AB^3 - A^3B}{\sqrt{2}}]$$

Consequently we have for the value of EH:

$$EH = \frac{L[2\sqrt{2}KB^2M - 2(2 + \sqrt{2})ABKM + AB^3\sqrt{2} - (2 + \sqrt{2})A^2B^2]\frac{v^2}{c^2}}{AB\sqrt{2} + 2AB^2\frac{v}{c} + \frac{v^2}{c^2}[(2 + \sqrt{2})(A^2B^2 + ABKM) + B^2KM\sqrt{2} + \frac{3AB^3 - A^3B}{\sqrt{2}}]}$$

and dividing to obtain the quotient:

$$EH = L[\frac{2KBM}{A} - (2\sqrt{2} + 2)KM + B^2 - (1 + \sqrt{2})AB]\frac{v^2}{c^2} \qquad \text{(Eq. 50)}$$

Substitution from equations 49 and 36 into equation 37, followed by subsequent reduction, results in:

$$FE = [1 - \frac{B}{\sqrt{2}}\frac{v}{c} + (\frac{A^2}{4} - \frac{B^2}{2} - \frac{2BKM}{A})\frac{v^2}{c^2}] L \qquad \text{(Eq. 51)}$$

QE was given by equation 40, and substitution of the value of FD from equation 36, followed by subsequent reduction, results in the following value of QE:

$$QE = L[\frac{v}{c} + (\frac{B}{\sqrt{2}} + \frac{2\sqrt{2}\ KM}{A})\frac{v^2}{c^2}] \qquad \text{(Eq. 52)}$$

EO' was given by equation 42, and substituting the value of EH from equation 50 into equation 42, (remembering that only terms of the order of $\frac{v^2}{c^2}$ or lower need be retained) we obtain:

$$EO' = \frac{\sqrt{2}\ L}{(1 + \sqrt{2})A - B}[\frac{2KBM}{A} - 2(1 + \sqrt{2})KM + B^2 - (1 + \sqrt{2})AB]\frac{v^2}{c^2} \qquad \text{(Eq. 53)}$$

We wish to know the value of HO', so we refer again to triangle HEO', where we have:

$$\frac{EH}{HO'} = \frac{\sin EO'H}{\sin HEO'} = \frac{\sin(\frac{\pi}{8} + \theta)}{\sin(\frac{3}{4}\pi - \theta - \phi_1'')}$$

Consequently:

$$HO' = \frac{\sin(\frac{\pi}{4} + \theta + \phi_1'')}{\sin(\frac{\pi}{8} + \theta)}\ EH \qquad \text{(Eq. 54)}$$

By the expansion of these functions, and the substitution of numerical values for the functions of $\frac{\pi}{4}$ and $\frac{\pi}{8}$, is obtained:

$$HO' = \sqrt{2 + \sqrt{2}}\ [\frac{A + \frac{v}{c}\frac{AB}{\sqrt{2}} + \frac{v^2}{c^2}(-\frac{A^3}{4} + AB^2 + BKM)}{\cos\theta + (1 + \sqrt{2})\sin\theta}]\ EH$$

Since EH is a function of $\frac{v^2}{c^2}$, it will be necessary to keep in

the numerator of the above only those terms which are of zero order in $\frac{v}{c}$. Substituting from equation 50 the value of EH, and subsequently reducing the result, we have:

$$HO = \frac{2A\sqrt{2 + \sqrt{2}}\ L}{(2 + \sqrt{2})A - B\sqrt{2}}\left[\frac{2KBM}{A} - 2(1 + \sqrt{2})KM + B^2 - (1 + \sqrt{2})AB\right]\frac{v^2}{c^2}$$

(Eq. 55)

The length of the return path of the ray from mirror A, namely, FH, can now be written as the sum of FE(equation 51) and EH (equation 50) as:

$$FH = L\left[1 - \frac{B}{\sqrt{2}}\frac{v}{c} + \frac{v^2}{c^2}\left(\frac{A^2}{4} + \frac{B^2}{2} - 2(1 + \sqrt{2})KM - (1 + \sqrt{2})AB\right)\right]$$

(Eq. 56)

9. LENGTH OF PATH FROM O TO MIRROR B, AND FROM MIRROR B BACK TO PLATE.

We shall refer now to plate 8. The incident ray at O divides, and a part of it passes through the plate, making an angle equal to ε with the x axis. Denote by t' the length of time required for the passage of this ray from the plate to mirror B. During this time the mirror will move from position B' to position B. If c equals the velocity of light and v the velocity of drift, then $B'B = vt'$ and $OB = ct'$. Hence we have:

$$\frac{OB}{B'B} = \frac{ct'}{vt'} \qquad \text{or} \qquad OB = \frac{c}{v}B'B$$

$$\angle B'BO = \theta - \varepsilon$$

$$\frac{L}{OB} = \frac{\sin(\theta - \varepsilon)}{\sin(180^\circ - \theta)}$$

$$OB = \frac{L \sin \theta}{\sin(\theta - \varepsilon)}$$

Expanding out this sine function, and replacing the functions of ε by means of equation 2, we obtain:

$$OB = \frac{L}{1 - \frac{v}{c} \cos \theta - \frac{v^2}{c^2} \frac{\sin^2 \theta}{2}}$$

Dividing this out we obtain the quotient:

$$OB = L[1 + \frac{v}{c} \cos \theta + \frac{v^2}{c^2} \frac{\cos^2 \theta + 1}{2}] \qquad \text{(Eq. 57)}$$

$$\text{or } OB = L[1 + \frac{v}{c} \frac{A + B}{2} + \frac{v^2}{c^2} \frac{3A^2 + 2AB + 3B^2}{8}] \qquad \text{(Eq. 58)}$$

The plan for finding the length of the return path from mirror B back to the plate, shown as BT in plate 8, is as follows: Resolve v into two components, one parallel to and the other perpendicular to the returning ray. Let RS = vt = the amount the plate moves while the ray is returning. Then RW represents the component of motion perpendicular to the ray and WS represents the component parallel to the ray. If there were no motion of the plate, the returning ray would strike the plate at J. But, due to the motion, the actual point of encounter is at T. Hence, the path of the returning ray is shortened by the amount JT. It can be seen that JT = CS, and that CS consists of two segments, CW + WS. It is also evident that the perpendicular component of the motion of the plate, RW, has the effect of shortening the path of the returning ray by the amount CW, while WS, the component

of motion parallel to the ray, shortens the return path by an additional amount equal to itself. This change in the position of the plate takes place in exactly the same length of time as that required by the return trip of the ray, BT.

Let OR, plate 8, equal the distance travelled by the plate while the ray from the plate at O is going to the mirror at B, and let t' equal the time required for this passage. Then:

$$OR = vt' \qquad \text{and} \qquad OB = ct'$$

Let t equal the time required for the passage of the ray from B to T, the same time interval as that required for the motion of the plate from R to S. Then:

$$RS = vt \qquad \text{and} \qquad BT = ct$$

Taking horizontal and vertical (x and y) axes through O, let us find the co-ordinates of B. We shall denote the co-ordinates of B as (x_1, y_1)

$$x_1 = OB \cos \varepsilon$$

The value of OB is given by equation 57, and cos ε may be found from equation 2, so that we have:

$$x_1 = L\left[1 + \frac{v}{c} \cos \theta + \frac{v^2}{c^2} \cos^2 \theta\right] \qquad \text{(Eq. 59)}$$

Similarly:

$$y_1 = - OB \sin \varepsilon = -L\left[\frac{v}{c} \sin \theta + \frac{v^2}{c^2} \sin \theta \cos \theta\right] \qquad \text{(Eq. 60)}$$

Using the same axes, let us call the co-ordinates of R, (x_2, y_2) and proceed to find them.

$$\frac{OR}{OB} = \frac{vt'}{ct'} \qquad \text{or} \qquad OR = \frac{v}{c}(OB)$$

Hence: $OR = L[\frac{v}{c} + \frac{v^2}{c^2} \cos\theta]$ (Eq. 61)

and the co-ordinates of R will be:

$$x_2 = OR \cos\theta = L[\frac{v}{c} \cos\theta + \frac{v^2}{c^2} \cos^2\theta] \quad \text{(Eq. 62)}$$

$$y_2 = -OR \sin\theta = -L[\frac{v}{c} \sin\theta + \frac{v^2}{c^2} \sin\theta \cos\theta] \quad \text{(Eq. 63)}$$

We note here that $y_1 = y_2$, thus showing that RB is parallel to the axis B of the interferometer, and since both OR and B'B make an angle θ with this axis, BR = L.

Then, in the triangle JRB:

$BR = L$

$\angle RBJ = \phi'_2$

$\angle JRB = \frac{7}{8}\pi$

$\angle RJB = \frac{\pi}{8} - \phi'_2$

$$\frac{BR}{\sin RJB} = \frac{BJ}{\sin JRB}$$

Therefore: $$BJ = L \frac{\sin\frac{\pi}{8}}{\sin(\frac{\pi}{8} - \phi'_2)} = \frac{L}{\cos\phi'_2 - \cot\frac{\pi}{8}\sin\phi'_2}$$

Substituting a numerical value for $\cot\frac{\pi}{8}$, and evaluating the functions of ϕ'_2 by means of equation 17 (note that ϕ'_2 is a small angle, hence $\tan\phi'_2 = \sin\phi'_2 = \phi'_2$) and finally dividing the denominator into unity to obtain the quotient,

$$BJ = L[1 + \frac{v}{c}(1 + \sqrt{2})\sin\theta + \frac{v^2}{c^2}(\frac{7 + 4\sqrt{2}}{2} \sin^2\theta + (2 + 2\sqrt{2}) \sin\theta \cos\theta)] \quad \text{(Eq. 64)}$$

We shall now resolve RS into two components, one parallel to and one perpendicular to the returning ray. The angle between RS and the returning ray is $(\theta + \phi'_2)$. The component parallel to the ray is:

$WS = RS \cos(\theta + \phi'_2) = vt \cos(\theta+\phi'_2)$

Expanding this cosine function and evaluating the functions of ϕ'_2, we get:

$$WS = vt[\cos\theta - \frac{v}{c}\sin^2\theta - \frac{v^2}{c^2}(\frac{5\sin^2\theta\cos\theta}{2})] \quad \text{(Eq. 65)}$$

The component perpendicular to the ray is RW.

$RW = vt \sin(\theta + \phi'_2)$ which becomes:

$$RW = vt[\sin\theta + \frac{v}{c}\sin\theta\cos\theta + \frac{v^2}{c^2}(\frac{5\sin\theta\cos^2\theta - \sin\theta}{2})] \quad \text{(Eq. 66)}$$

In order to find CW, we need to know the following angles:

$$RCW = \frac{\pi}{8} - \phi'_2$$

$$CWR = 90^\circ$$

$$CRW = \frac{3}{8}\pi + \phi'_2$$

$CW = RW \cot(\frac{\pi}{8} - \phi'_2)$, which by equation 66 becomes:

$$CW = vt[\sin\theta + \frac{v}{c}\sin\theta\cos\theta + \frac{v^2}{c^2}(\frac{5\sin\theta\cos^2\theta - \sin\theta}{2}) \cot(\frac{\pi}{8} - \phi'_2) \quad \text{(Eq. 67)}$$

In order to simplify equation 67, it will be necessary to expand this cotangent function into a quotient, divide this

quotient out and obtain a series in powers of $\frac{v}{c}$, and finally to multiply this series by the factor shown above, to obtain the value of CW. This work, while lengthy and of such a nature that it must be done carefully to avoid errors, is nevertheless straightforward, and yields the result:

$$CW = vt[(\sqrt{2} + 1)\sin\theta + \sin\theta \frac{v}{c}(2(2 + \sqrt{2}) \sin\theta + (1 + \sqrt{2})\cos\theta) + \sin\theta \frac{v^2}{c^2}(6(2 + \sqrt{2})\sin\theta \cos\theta + 2(4 + 3\sqrt{2})\sin^2\theta + \frac{(\sqrt{2} + 1)(5\cos^2\theta - 1)}{2})] \qquad \text{(Eq. 68)}$$

As may be seen in plate 8, JT = CS = CW + WS and since WS is given by equation 65, we may write:

$$CS = vt[(\sqrt{2} + 1)\sin\theta + \cos\theta + \frac{v}{c}[(2\sqrt{2} + 3)\sin^2\theta + (1 + \sqrt{2})\sin\theta \cos\theta] + \frac{v^2}{c^2}[2(1 + \sqrt{2})\sin\theta + \frac{19 + 12\sqrt{2}}{2}\sin^2\theta \cos\theta + \frac{11 + 7\sqrt{2}}{2}\sin^3\theta]] \qquad \text{(Eq. 69)}$$

Now, BT + TJ = BJ. As already stated, BT = ct, TJ = CS, which is given above in equation 69, and BJ is given in equation 64. Consequently the above becomes:

$$ct + vt[(\sqrt{2} + 1)\sin\theta + \cos\theta + \frac{v}{c}((2\sqrt{2} + 3)\sin^2\theta + (1 + \sqrt{2})\sin\theta \cos\theta) + \frac{v^2}{c^2}(2(1 + \sqrt{2})\sin\theta + \frac{19 + 12\sqrt{2}}{2}\sin^2\theta \cos\theta + \frac{11 + 7\sqrt{2}}{2}\sin^3\theta)] = (1 + \frac{v}{c}\sin\theta(1 + \sqrt{2}) + \frac{v^2}{c^2}[\frac{7 + 4\sqrt{2}}{2}\sin^2\theta + (2 + 2\sqrt{2})\sin\theta \cos\theta]) L \qquad \text{(Eq. 70)}$$

The above equation solved for t, gives the length of time required for the passage of the ray from B to T, and of the movement of the plate from R to S. Solving equation 70, we have:

$$t = \frac{l}{c}[1 - \frac{v}{c}\cos\theta + \frac{v^2}{c^2}(\frac{\sin^2\theta}{2} + \cos^2\theta + 2(1 + \sqrt{2})\sin\theta\cos\theta)]L \qquad \text{(Eq. 71)}$$

From this follows the results:

$$BT = ct = L[1 - \frac{v}{c}\cos\theta + \frac{v^2}{c^2}(\frac{\sin^2\theta}{2} + \cos^2\theta + 2(1 + \sqrt{2})\sin\theta\cos\theta)]$$

$$RS = vt = L[\frac{v}{c} - \frac{v^2}{c^2}\cos\theta]$$

These results, in terms of A and B (equations 29) become:

$$BT = L[1 - \frac{v}{c}\frac{A + B}{2} + \frac{v^2}{c^2}\frac{7A^2 - B^2 + 2AB + 4\sqrt{2}A^2 - 4\sqrt{2}B^2}{8}] \qquad \text{(Eq.72)}$$

$$RS = L[\frac{v}{c} - \frac{v^2}{c^2}\frac{A + B}{2}] \qquad \text{(Eq. 73)}$$

10. EQUATIONS OF RAYS IN THEIR FINAL PATHS, AND CO-ORDINATES OF THEIR POINT OF INTERSECTION

The next step in the solution of the problem is the finding of the equations of the two rays in their final tracks, the one as it is reflected from the plate at B, and the other as it is reflected from the mirror A a t F, and passes through

the plate at H. We shall just find the co-ordinates of point F, designating them as (x_f, y_f). Again, let us assume a set of rectangular axes with the origin at O, plate 6, with the x axis coinciding with arm B of the interferometer at the instant the ray from the source is incident at O. With these assumptions, we have:

$x_f = OF \cos(\frac{\pi}{4} - \delta')$, which, by means of equations 30 and 35, becomes:

$$x_f = \frac{L}{\sqrt{2}}[1 + \frac{v}{c}\frac{A + B}{\sqrt{2}} + \frac{v^2}{c^2}(KA^2 + 2KAB - KB^2 + \frac{AB + B^2}{2})] \quad \text{(Eq. 74)}$$

$y_f = OF \sin(\frac{\pi}{4} - \delta')$ which reduces to:

$$y_f = \frac{L}{\sqrt{2}}[1 + \frac{B - A}{\sqrt{2}}\frac{v}{c} - \frac{v^2}{c^2}(KA^2 + 2KAB - KB^2 + \frac{AB - B^2}{2})] \quad \text{(Eq. 75)}$$

The reflected ray from F makes an angle with the x axis equal to $(\frac{\pi}{4} + \phi''_1)$ (see plate 7). Hence the slope of this line is $\tan(\frac{\pi}{4} + \phi''_1)$ which may be expanded into a quotient, and divided out to obtain:

$$\tan(\frac{\pi}{4} + \phi''_1) = 1 + \frac{2A}{\sqrt{2}}\frac{v}{c} + 2(AB + A^2K + 2KAB - KB^2 + \frac{A^2}{2})\frac{v^2}{c^2}$$

(Eq. 76)

Writing the equation of the line through F having this slope in the well known point slope form,

$y - y_f = \tan(\frac{\pi}{4} + \phi''_1)[x - x_f]$ and reducing the result, we obtain

$$x[1 + A\sqrt{2}\frac{v}{c} + \frac{v^2}{c^2}\,2(AB + KA^2 + 2KAB - KB^2 + \frac{A^2}{2})] - y = 2L[A\frac{v}{c} +$$

$$\frac{v^2}{c^2}(\sqrt{2}AB + \sqrt{2}KA^2 + \frac{A^2}{\sqrt{2}} + 2\sqrt{2}\,KAB - \sqrt{2}\,KB^2)]$$

(Eq. 77)

We shall now find the co-ordinates of T, and the equation of the ray which is reflected from the plate at T. We denote the co-ordinates of T as (x_t, y_t), and remembering that we have already made use of the co-ordinates of B, denoting them as (x_1, y_1) we have: (see plate 8)

$x_t = x_1 - BT \cos \phi'_2$

x_1 is given by equation 59, BT is given by equation 72, and $\cos \phi'_2$ may be found from equation 17.

Likewise, $y_t = y_1 - BT \sin \phi'_2$, which may be evaluated by means of the above mentioned equations and equation 60. Carrying out the necessary substitutions, we get:

$$x_t = L[2 \cos \theta \frac{v}{c} - \frac{v^2}{c^2}(2 + 2\sqrt{2})\sin \theta \cos \theta] \qquad \text{(Eq. 78)}$$

$$y_t = -L[2 \sin \theta \frac{v}{c} + 2 \frac{v^2}{c^2} \sin \theta \cos \theta] \qquad \text{(Eq. 79)}$$

By referring to plate 5, it may be seen that the ray which is reflected from T makes an angle $(\frac{5}{8}\pi - \phi''_2)$ with the x axis. Consequently, the slope of this ray is given by the tangent of this angle. Substituting in the above expression the value of ϕ''_2 given by equation 23, it becomes the angle $(\frac{\pi}{4} - \delta)$,

and by suitable reduction, the slope of the ray becomes:

$$\tan(\frac{\pi}{4} - \delta) = 1 + \sqrt{2}A\frac{v}{c} + \frac{v^2}{c^2}[B^2 + 2K(B^2 - A^2) - 4KAB]$$

The equation of the ray through T, making the angle $(\frac{\pi}{4} - \delta)$ with the x axis, can then be written as:

$$x[1 + A\sqrt{2}\,\frac{v}{c} + \frac{v^2}{c^2}(B^2 + 2K(B^2 - A^2) - 4KAB)] - y = 2L[(\cos\theta + \sin\theta)\frac{v}{c} + \frac{v^2}{c^2}(A\sqrt{2}\cos\theta - \sqrt{2}\sin\theta\cos\theta)] \qquad \text{(Eq. 80)}$$

By making use of our substitutions for sin θ and cos θ in terms of A and B, as outlined in equation 29, we may write the equations of the two rays (equations 77 and 80), as follows:

ray through T:

$$x[1 + A\sqrt{2}\,\frac{v}{c} + \frac{v^2}{c^2}(B^2 + 2KB^2 - 2KA^2 - 4KAB)] - y = 2L[A\frac{v}{c} + \frac{v^2}{c^2}\frac{\sqrt{2}(A^2 + 2AB + B^2)}{4}] \qquad \text{(Eq. 81)}$$

ray through F:

$$x[1 + A\sqrt{2}\,\frac{v}{c} + \frac{v^2}{c^2}(-2KB^2 + 2KA^2 + 2AB + A^2 + 4KAB)] = 2L[A\frac{v}{c} + \frac{v^2}{c^2}(\sqrt{2}AB + \sqrt{2}KA^2 + \frac{A^2}{\sqrt{2}} + 2\sqrt{2}KAB - \sqrt{2}KB^2)] + y \qquad \text{(Eq. 82)}$$

Subtracting equation 82 from equation 81 gives:

$$x = \frac{L}{\sqrt{2}} \qquad \text{(Eq. 83)}$$

and substituting this value of x in either of the above

equations will give for the y co-ordinate of the point of intersection:

$$y = L\left[\frac{1}{\sqrt{2}} - A\frac{v}{c} + \frac{v^2}{c^2}\,\frac{\sqrt{2}(2KB^2 - (1 + 2K)A^2 - 2(1 + 2K)AB)}{2}\right] \qquad \text{(Eq. 84)}$$

Let us denote this point of intersection of the two rays as the point Z, and designate it by the co-ordinates (x_o, y_o). Hence we would have:

$$x_o = \frac{L}{\sqrt{2}}, \qquad y_o = L\left[\frac{1}{\sqrt{2}} - A\frac{v}{c} + \frac{v^2}{c^2}\,\frac{\sqrt{2}(2KB^2 - (1 + 2K)A^2}{2} + \frac{-2(1 + 2K)AB)}{2}\right] \qquad \text{(Eq. 85)}$$

11. WAVE LENGTHS OF THE RAYS ALONG THE VARIOUS PATHS

It was stated early in this paper, and is proved in the appendix, that the law governing the reflection of light from a moving plane surface is as follows:

$$\tan\frac{\phi'}{2} = \frac{c + u}{c - u}\tan\frac{\phi}{2} \qquad \text{(Eq. 3)}$$

where ϕ' = the angle of reflection, ϕ = the angle of incidence, c = the velocity of light, and u = the velocity of the reflecting surface, positive when the surface moves away from the source.

Due to the fact that the angle of reflection suffers a slight change, the wave length of the reflected ray will also

be slightly altered. The law according to which this change takes place is given by:

$$\frac{\Sigma'}{\Sigma} = \frac{\sin \phi'}{\sin \phi} \quad (^1) \qquad \text{(Eq. 86)}$$

where Σ represents the wave length before reflection, ϕ, the angle of incidence, ϕ', the angle of reflection, and Σ', the wave length after reflection. Combining equations 3 and 86, we may derive the equation:

$$\Sigma' = \Sigma \frac{c^2 - u^2}{c^2 + u^2 - 2cu \cos \phi} \quad (^2) \qquad \text{(Eq. 87)}$$

We shall calculate first the wave length of the ray along the path OF from O to mirror A. In this case the angle of incidence $\phi = (\frac{3}{8}\pi - \frac{v}{c} \sin \theta)$ and the velocity of the plate perpendicular to itself $u = v \sin(\theta + \frac{\pi}{8})$. Substituting in equation 87, we have:

$$\Sigma_1' = \Sigma \frac{c^2 - v^2 \sin^2(\theta + \frac{\pi}{8})}{c^2 + v^2 \sin^2(\theta + \frac{\pi}{8}) - 2cv \sin(\theta + \frac{\pi}{8}) \cos(\frac{3}{8}\pi - \frac{v}{c} \sin \theta)}$$

where Σ_1' represents the wave length of the reflected ray, and Σ represents the fundamental wave length of the ray from the source. Dividing this out to obtain the quotient, we find that:

- -

(1) For proof of this law, see appendix C, page 76.

(2) For complete derivation of equation 87, see appendix D, page 77.

$$\Sigma'_1 = \Sigma[1 + 2\frac{v}{c}\sin(\theta + \frac{\pi}{8})\cos(\frac{3}{8}\pi - \frac{v}{c}\sin\theta) + 2\frac{v^2}{c^2}\sin^2(\theta + \frac{\pi}{8})$$

$$\cos(\frac{3}{4}\pi - 2\frac{v}{c}\sin\theta)]$$

which, upon expansion of the trigonometric functions and subsequent simplification, becomes

$$\Sigma'_1 = \Sigma[1 + \frac{v}{c}(\frac{\sin\theta}{\sqrt{2}} + \frac{2-\sqrt{2}}{2}\cos\theta) + \frac{v^2}{c^2}(\frac{\sin^2\theta}{2} + \frac{\sqrt{2}-2}{2}\sin\theta$$

$$\cos\theta + \frac{1-\sqrt{2}}{2}\cos^2\theta)]$$

and upon substitution from equations 29 this becomes:

$$\Sigma'_1 = \Sigma[1 + \frac{v}{c}\frac{A + B - \sqrt{2}B}{2} + \frac{v^2}{c^2}\frac{2B^2 - \sqrt{2}B^2 - \sqrt{2}AB}{4}] \quad \text{(Eq. 88)}$$

We shall next find the wave length along the path FH, from mirror A back to the plate. As may be seen from plate 3, the angle of incidence in this case is: $\phi'_1 - \frac{3}{8}\pi = \delta'$, and by equation 35,

$$\delta' = \frac{A}{\sqrt{2}}\frac{v}{c} + K(A^2 + 2AB - B^2)\frac{v^2}{c^2}$$

The velocity of the mirror in the direction perpendicular to itself is given by: $u = v\cos(\theta + \frac{\pi}{4})$. Therefore, if we represent by Σ''_1 the wave length along this path, we have:

$$\Sigma''_1 = \Sigma'_1 \frac{c^2 - v^2\cos^2(\theta + \frac{\pi}{4})}{c^2 + v^2\cos^2(\theta+\frac{\pi}{4}) - 2cv\cos(\theta+\frac{\pi}{4})\cos[\frac{A}{\sqrt{2}}\frac{v}{c} + K(A^2+2AB-B^2)\frac{v^2}{c^2}]}$$

The simplification of the above fraction leads to the result:

$$\Sigma_1'' = \Sigma_1'[1 + 2\frac{v}{c}\cos(\theta + \frac{\pi}{4}) + 2\frac{v^2}{c^2}\cos^2(\theta + \frac{\pi}{4})]$$

or: $$\Sigma_1'' = \Sigma_1'[1 + \sqrt{2}\,B\frac{v}{c} + B^2\frac{v^2}{c^2}] \qquad \text{(Eq. 89)}$$

It will be advantageous to have Σ_1'' expressed in terms of the fundamental wave length Σ, so substituting for Σ_1' in 89, we obtain:

$$\Sigma_1'' = \Sigma[1 + \frac{v}{c}\frac{A + B + \sqrt{2}\,B}{2} + \frac{v^2}{c^2}\frac{2B^2 + \sqrt{2}\,B^2 + \sqrt{2}\,AB}{4}] \qquad \text{(Eq. 90)}$$

Along the path OB, of course, the wave length remains the same as incident wave length Σ. Upon reflection from the mirror at B, however, there will be a change in wave length which we shall now calculate. As may be seen from plate 5, the incident wave length is in this case equal to Σ, while the velocity of the mirror perpendicular to itself is equal to $(v \cos\theta)$. Hence, denoting the reflected wave length by Σ_2', we have:

$$\Sigma_2' = \Sigma\frac{c^2 - v^2\cos^2\theta}{c^2 + v^2\cos^2\theta - 2cv\cos\theta\cos(\frac{v}{c}\sin\theta)}$$

This in turn will reduce to the expression:

$$\Sigma_2' = \Sigma[1 + \frac{2v}{c}\cos\theta + 2\frac{v^2}{c^2}\cos^2\theta]$$

In terms of A and B, this is:

$$\Sigma'_2 = \Sigma[1 + \frac{v}{c}(A + B) + \frac{v^2}{c^2}\frac{A^2 + 2AB + B^2}{2}] \qquad \text{(Eq. 91)}$$

The angle of incidence, when the ray is reflected from the plate at T, is $(\frac{3}{8}\pi + \phi'_2)$ and by equation 17,

$$\phi'_2 = \frac{v}{c}\sin\theta + 2\frac{v^2}{c^2}\sin\theta\cos\theta.$$

The component of the velocity of the plate perpendicular to itself is: $-v\sin(\theta + \frac{\pi}{8})$. (The negative sign is a result of the direction of motion toward the source). Calling the wave length of the ray after reflection Σ''_2, we have:

$$\Sigma''_2 = \Sigma'_2 \frac{c^2 - v^2\sin^2(\theta + \frac{\pi}{8})}{c^2 + v^2\sin^2(\theta+\frac{\pi}{8}) + 2cv\sin(\theta+\frac{\pi}{8})\cos[\frac{3}{8}\pi + \frac{v}{c}\sin\theta + 2\frac{v^2}{c^2}\sin\theta\cos\theta]}$$

Expanding the functions involved and obtaining the quotient, we get as the result:

$$\Sigma''_2 = \Sigma'_2[1 - \frac{v}{c}(\frac{\sqrt{2}}{2}\sin\theta + \cos\theta - \frac{\sqrt{2}}{2}\cos\theta) + \frac{v^2}{c^2}(\frac{1}{2}\sin^2\theta + \frac{\sqrt{2} - 2}{2}\sin\theta\cos\theta + \frac{1 - \sqrt{2}}{2}\cos^2\theta)$$

When transformed into an expression involving A and B, this becomes:

$$\Sigma''_2 = \Sigma'_2[1 - \frac{v}{c}\frac{A + B - \sqrt{2}\,B}{2} + \frac{v^2}{c^2}\frac{2B^2 - \sqrt{2}\,AB - \sqrt{2}\,B^2}{4}] \qquad \text{(Eq. 92)}$$

Expressing this in terms of the fundamental wave length Σ, by replacing Σ'_2 from equation 91, we have:

$$\Sigma''_2 = \Sigma[1 + \frac{v}{c}\frac{A + B + \sqrt{2}\,B}{2} + \frac{v^2}{c^2}\frac{2B^2 + \sqrt{2}\,AB + \sqrt{2}\,B^2}{4}] \quad \text{(Eq. 93)}$$

This, we see by comparison, is identical to the wave length Σ''_1 which was given by equation 90. This is an important result, as it shows that the two final rays, which will be gathered up and brought to a focus in order that we may observe the fringes formed, are of equal wave length.

12. CALCULATION OF THE NUMBER OF WAVE LENGTHS IN THE SEPARATE PATHS.

We shall refer now to plate 9, which shows the general nature of the paths followed by the two rays of light from the time they leave the plate at O, until they are recombined by a lens to produce the fringes. The various lengths of paths, and the wave lengths along these paths as has already been determined, are as follows:

$$OF = L[1 + \frac{B}{\sqrt{2}}\frac{v}{c} + \frac{A^2 + 2B^2}{4}\frac{v^2}{c^2}] \quad \text{(Eq. 30)}$$

$$FH = L[1 - \frac{B}{\sqrt{2}}\frac{v}{c} + \frac{v^2}{c^2}(\frac{A^2}{4} + \frac{B^2}{2} - 2(1 + \sqrt{2})KM - (1 + \sqrt{2})AB)] \quad \text{(Eq. 56)}$$

$$OB = L[1 + \frac{v}{c}\frac{A + B}{2} + \frac{v^2}{c^2}\frac{3A^2 + 2AB + 3B^2}{8}] \quad \text{(Eq. 58)}$$

$$BT = L[1 - \frac{v}{c}\frac{A + B}{2} + \frac{v^2}{c^2}\frac{7A^2 - B^2 + 2AB + 4\sqrt{2}\,A^2 - 4\sqrt{2}\,B^2}{8}] \quad \text{(Eq. 72)}$$

Wave length along OF:

$$\Sigma'_1 = \Sigma\left[1 + \frac{v}{c}\frac{A + B - \sqrt{2}B}{2} + \frac{v^2}{c^2}\frac{2B^2 - \sqrt{2}\,B^2 - \sqrt{2}\,AB}{4}\right] \quad \text{(Eq. 88)}$$

Wave length along FH:

$$\Sigma''_1 = \Sigma\left[1 + \frac{v}{c}\frac{A + B + \sqrt{2}\,B}{2} + \frac{v^2}{c^2}\frac{2B^2 + \sqrt{2}\,AB + \sqrt{2}\,B^2}{4}\right] \quad \text{(Eq. 90)}$$

Wave length along OB = Σ

Wave length along BT:

$$\Sigma'_2 = \Sigma\left[1 + \frac{v}{c}(A + B) + \frac{v^2}{c^2}\frac{A^2 + 2AB + B^2}{2}\right] \quad \text{(Eq. 91)}$$

Wave length after reflection from T:

$$\Sigma''_2 = \Sigma\left[1 + \frac{v}{c}\frac{A + B + \sqrt{2}\,B}{2} + \frac{v^2}{c^2}\frac{2B^2 + \sqrt{2}\,AB + \sqrt{2}\,B^2}{4}\right] \quad \text{(Eq. 93)}$$

Now, the actual number of wave lengths along any of these paths is, of course, given by the result of dividing the length of the path by the wave length along that path. Hence, denoting the number of wave lengths along the path OF by N_1, we have:

$$N_1 = \frac{OF}{\Sigma'_1} = \frac{L}{\Sigma}\left[1 + \frac{v}{c}\frac{2\sqrt{2}\,B - A - B}{2} + \frac{v^2}{c^2}\frac{2A^2 + 2AB - 2\sqrt{2}\,AB + 5B^2 - 2\sqrt{2}\,B^2}{4}\right] \quad \text{(Eq. 94)}$$

Let N_2 = number of wave lengths along path FH.

$$N_2 = \frac{FH}{\Sigma''_1} = \frac{L}{\Sigma}\left[1 - \frac{v}{c}\frac{A + B + 2\sqrt{2}\,B}{2} + \frac{v^2}{c^2}\left(\frac{A^2}{2} - 2(1 + \sqrt{2})KM - \frac{AB}{2} - \frac{\sqrt{2}\,AB}{2} + \frac{5B^2}{4} + \frac{\sqrt{2}\,B^2}{2}\right)\right] \quad \text{(Eq. 95)}$$

Let N_3 = number of wave lengths along path OB

$$N_3 = \frac{L}{\Sigma}\left[1 + \frac{v}{c}\frac{A + B}{2} + \frac{v^2}{c^2}\frac{3A^2 + 2AB + 3B^2}{8}\right] \qquad \text{(Eq. 96)}$$

Let N_4 = number of wave lengths along path BT.

$$N_4 = \frac{L}{\Sigma}\left[1 - \frac{v}{c}\frac{3(A + B)}{2} + \frac{v^2}{c^2}\frac{15A^2 + 18AB + 7B^2 + 4\sqrt{2}\,A^2 - 4\sqrt{2}\,B^2}{8}\right]$$

(Eq. 97)

The total number of wave lengths along the path OFH, is:

$$N_1 + N_2 = \frac{L}{\Sigma}\left[2 - (A + B)\frac{v}{c} + \frac{v^2}{c^2}\left(A^2 - \sqrt{2}\,AB + \frac{5}{2}B^2 - 2(1 + \sqrt{2})KM\right)\right]$$

(Eq. 98)

The total number along the path OBT is:

$$N_3 + N_4 = \frac{L}{\Sigma}\left[2 - (A + B)\frac{v}{c} + \frac{v^2}{c^2}\left(\frac{9A^2 + 10AB + 5B^2 + 2\sqrt{2}\,A^2 - 2\sqrt{2}\,B^2}{4}\right)\right]$$

If we subtract the number of wave lengths in the path OFH from the number in the path OBT, we obtain:

$$(N_3 + N_4) - (N_1 + N_2) = \frac{v^2}{c^2}\frac{L}{\Sigma}\left[\frac{5A^2}{4} + \frac{5AB}{2} + \sqrt{2}\,AB - \frac{5B^2}{4} + \frac{\sqrt{2}\,A^2}{2} - \frac{\sqrt{2}\,B^2}{2} + 2(1 + \sqrt{2})KM\right] \qquad \text{(Eq.99)}$$

Equation 99 gives the actual number of wave lengths by which the path OBT exceeds the path OFH. Since the fundamental wave length Σ has been used, this is a measure of the amount by which path OBT exceeds path OFH.

13. ACTUAL DIFFERENCE IN PATH, MEASURED IN TERMS OF Σ, THE FUNDAMENTAL WAVE LENGTH, FROM O TO A LENS EQUALLY INCLINED TO THE TWO RAYS IN THEIR FINAL PATHS.

The co-ordinates of Z a s already found, (x_o, y_o) are:

$$\left(\frac{L}{\sqrt{2}},\ L\left[\frac{1}{\sqrt{2}} - A\frac{v}{c} + \frac{v^2}{c^2}\,\frac{2\sqrt{2}\,KB^2 - \sqrt{2}(1 + 2K)A^2 - 2\sqrt{2}(1 + 2K)AB}{2}\right]\right)$$

(Eq. 85)

The co-ordinates of T are:

$$x_t = L\left[(A + B)\frac{v}{c} - \frac{v^2}{c^2}\,\frac{A^2 - B^2 + \sqrt{2}\,A^2 - \sqrt{2}\,B^2}{2}\right] \quad \text{(Eq. 78)}$$

$$y_t = -L\left[(A - B)\frac{v}{c} + \frac{v^2}{c^2}\,\frac{A^2 - B^2}{2}\right] \quad \text{(Eq. 79)}$$

and the co-ordinates of F are:

$$x_f = \frac{L}{\sqrt{2}}\left[1 + \frac{v}{c}\,\frac{A + B}{\sqrt{2}} + \frac{v^2}{c^2}\left(KA^2 + 2KAB - KB^2 + \frac{AB + B^2}{2}\right)\right] \quad \text{(Eq. 74)}$$

$$y_f = \frac{L}{\sqrt{2}}\left[1 + \frac{v}{c}\,\frac{B - A}{\sqrt{2}} - \frac{v^2}{c^2}\left(KA^2 + 2KAB - KB^2 + \frac{AB - B^2}{2}\right)\right] \quad \text{(Eq. 75)}$$

We shall now proceed to find FZ.

$$FZ = \sqrt{(x_f - x_o)^2 + (y_f - y_o)^2}$$

While the details of this computation are fairly lengthy, all the required information ha s been set forward in the preceding pages. The computation leads to the result:

$$FZ = L\left[\frac{v}{c}\,\frac{A + B}{\sqrt{2}} + \frac{v^2}{c^2}\,\frac{A^2 + 2AB + B^2 + 2KM}{2}\right] \quad \text{(Eq. 100)}$$

$$FH = L[1 - \frac{B}{\sqrt{2}}\frac{v}{c} + \frac{v^2}{c^2}(\frac{A^2}{4} + \frac{B^2}{2} - 2KM - 2\sqrt{2}KM - AB - \sqrt{2}\,AB)]$$

(Eq. 56)

Subtracting FZ from FH, we get:

$$ZH = L[1 - \frac{A + 2B}{\sqrt{2}}\frac{v}{c} - \frac{v^2}{c^2}(\frac{A^2}{4} + 3KM + 2\sqrt{2}\,KM + 2AB + \sqrt{2}\,AB)]$$

(Eq. 101)

We shall now find ZT by means of the equation:

$$ZT = \sqrt{(x_o - x_t)^2 + (y_o - y_t)^2}$$

This computation, likewise, although straightforward, is lengthy. However, it leads to the result:

$$ZT = L[1 - \frac{A + 2B}{\sqrt{2}}\frac{v}{c} + \frac{v^2}{c^2}(\frac{A^2}{4} - \frac{B^2}{2} + \frac{A^2}{\sqrt{2}} - \frac{B^2}{\sqrt{2}} - AB - KM)] \quad \text{(Eq. 102)}$$

Subtracting ZH from ZT, we obtain:

$$ZT - ZH = L\,\frac{v^2}{c^2}[\frac{A^2}{2} - \frac{B^2}{2} + \frac{A^2}{\sqrt{2}} - \frac{B^2}{\sqrt{2}} + 2KM + 2\sqrt{2}\,KM + \sqrt{2}\,AB + AB)]$$

(Eq. 103)

Hence, the length ZT exceeds the length ZH by the amount shown as the right hand member of equation 103. This length, divided by the wave length along ZH, represents the amount (measured in wave lengths) by which the path ZH must be increased in order to find a point along this path which shall be the same distance from Z as is the point T. This is necessary in order to insure that a lens, placed in the final path of the rays, will be equally inclined to both ZH and ZT. The wave length along ZH is given by equation 90, as Σ''_1 and

calling this small amount by which the path ZH must be increased N_o, we have:

$$N_o = \frac{ZT - ZH}{\Sigma_1''} = \frac{L}{\Sigma}\frac{v^2}{c^2}\left(\frac{A^2}{2} - \frac{B^2}{2} + \frac{A^2}{\sqrt{2}} - \frac{B^2}{\sqrt{2}} + 2KM + 2\sqrt{2}\,KM + \sqrt{2}\,AB + AB\right) \qquad \text{(Eq. 104)}$$

Since we have already learned that the path OBT exceeds the path OFH by: (see equation 99)

$$N_3 + N_4 - N_1 - N_2 = \frac{v^2}{c^2}\frac{L}{\Sigma}\left[\frac{5A^2}{4} + \frac{5AB}{2} + \sqrt{2}\,AB - \frac{5B^2}{4} + \frac{\sqrt{2}\,A^2}{2} - \frac{\sqrt{2}\,B^2}{2} + 2(1 + \sqrt{2})KM\right]$$

we shall now subtract from this the amount N_o:

$$(N_3 + N_4 - N_1 - N_2) - N_o = \frac{3A^2 + 6AB - 3B^2}{4}\frac{v^2}{c^2}\frac{L}{\Sigma} \qquad \text{(Eq. 105)}$$

to obtain the actual measure of the difference in the length of the path followed by the two halves of any ray which may be incident at O.

If, in the right hand member of equation 105, we replace A and B from equations 29, we obtain:

$$(N_3 + N_4 - N_1 - N_2) - N_o = \frac{3}{2}\frac{v^2}{c^2}\frac{L}{\Sigma}(\sin 2\theta + \cos 2\theta) \qquad \text{(Eq. 106)}$$

CONCLUSION

The conclusion to be drawn from the foregoing pages is that if an interferometer is constructed with its two arms making an angle of forty-five degrees instead of the usual ninety degrees, the expected maximum displacement of the fringes should be more than twice as great as the maximum displacement shown by an interferometer of similar size with perpendicular arms.

The actual fringe shift depends upon the effective difference in optical path, measured in terms of wave lengths along the respective component paths, from the time the ray is first divided at the half silvered plate until the two component rays are brought to the point of entering a lens placed in the final path of the rays so adjusted as to allow the rays to enter the lens equally inclined to its axis. The relation is such that if the fringes are located for any particular orientation of the instrument, and the instrument is then turned through an angle such that now the length of the path along one arm changes relative to the length of the path along the other arm, this change in the relative lengths of the paths will result in a shift in the positions of the fringes. Furthermore, the amount of displacement of the fringes will be exactly equal in terms of the distance between fringes to the change in the relative lengths of the two paths measured in wave lengths of the light used.

If θ represents the angle between the principal, or source

axis of the interferometer and the direction of drift of the earth, the following equation gives the effective path difference directly in wave lengths:

$$\text{path difference} = \frac{3}{2}\frac{V^2}{C^2}\frac{l}{\lambda}(\sin 2\theta + \cos 2\theta)$$

A brief analysis of the above equation shows that the measure of the path difference, commonly called the "effect", is doubly periodic with respect to the angle of drift, θ. For setting the right-hand side equal to zero, we obtain: $\tan 2\theta = -1$ from which we have the following values of θ: $67\frac{1}{2}^\circ$, $157\frac{1}{2}^\circ$, $247\frac{1}{2}^\circ$, and $337\frac{1}{2}^\circ$. As is shown in plate 10-A, these angles place the instrument in the four positions which make the arms symmetrical with respect to the drift. Next, differentiating the right-hand side with respect to θ and setting the derivative equal to zero, we obtain as the angles for maximum or minimum effect $\theta = 22\frac{1}{2}^\circ$, $112\frac{1}{2}^\circ$, $202\frac{1}{2}^\circ$, and $292\frac{1}{2}^\circ$. As may be easily seen by inspection, the angles $22\frac{1}{2}^\circ$ and $202\frac{1}{2}^\circ$ each give maximum values (positive) to the path difference function, while $112\frac{1}{2}^\circ$ and $292\frac{1}{2}^\circ$ result in minimum values. Thus, for one complete rotation of the interferometer, the effect passes through two maxima, two minima, and four zero values, which means that it is doubly periodic. This is to be expected, since, as plate 10-A shows, regardless of the angle between the arms, there will be four positions of the instrument which will cause the arms to occupy symmetric positions with respect to the drift so that their optical paths will be equal.

The maximum difference in path is of course only one-half of the maximum fringe shift to be expected since, as the interferometer is rotated, first one arm has the longer path, and then the other, which causes the fringes to shift first in one direction and then in the opposite, the maximum displacement being the arithmetic sum of these two displacements. Evaluating the formula found for angles giving the greatest shift in either direction, we obtain as the actual expression for the total fringe displacement: $3\sqrt{2}\ \frac{l}{\lambda}\frac{V^2}{C^2}$.

Comparing this with the effect which is to be expected from an interferometer whose arms are perpendicular, we take this value from D. C. Miller's paper :* total fringe displacement is $2\ \frac{l}{\lambda}\frac{V^2}{C^2}$. This value is the one used by Miller to reduce all his observed shifts to relative velocities between the earth and ether. A comparison of the two values shows that the former is more than twice as great as the latter, and plate 10-C shows graphically how the expected shifts would compare in magnitude. This plate also shows the phase relationship between the two cases. Another comparison may be made on the basis of the interferometer actually used by Miller in his Mount Wilson experiments. Here the arms were such that $\frac{l}{\lambda}$ equaled 112,000,000, and with the arms perpendicular should have produced a shift of 1.12 fringes. With the arms at an angle of forty-five degrees, the expected shift would amount to 2.37 fringes.

* Reviews of Modern Physics 5, 227 (1933)

The formula of W. B. Cartmel was mentioned in the introduction. Mr. Cartmel gave an equation for finding the fringe shift as follows:

$$F = 2 \frac{l}{\lambda} \frac{V^2}{C^2} \sin \Omega \sin (2 \theta \pm \Omega)$$

where Ω is equal to ninety degrees minus the angle between the end mirror normals. For the case where the arms are inclined at forty-five degrees, then, this becomes:

$$F = \frac{l}{\lambda} \frac{V^2}{C^2} (\sin 2 \theta + \cos 2 \theta)$$

which is only two-thirds as great a shift as that shown by the equation which the author derived.

In the introduction the author mentioned an apparent difficulty in Cartmel's equation. According to his definition of Ω, if the two arms are taken parallel to each other, Ω becomes ninety degrees, and his expression for the shift becomes:

$$F = \frac{2l}{\lambda} \frac{V^2}{C^2} \sin 90^\circ \sin (2 \theta \pm 90^\circ) \quad \text{or}$$

$$F = 2 \frac{l}{\lambda} \frac{V2}{C^2} \cos 2 \theta$$

It appears as if an effect is predicted when the two arms are taken parallel to each other, which does not seem reasonable.

Briefly, then, the contents of this paper indicate that the efficiency of an interferometer when used in connection with the ether drift experiment is more than doubled by making the arms inclined with respect to each other, forty-five degrees instead of ninety degrees.

At this time the author wishes to express his sincere appreciation for the inspiring guidance and the numerous helpful suggestions received from Dr. Royal M. Frye, under whose direction this problem has been carried out.

COMPREHENSIVE DIGEST

This dissertation investigates the theory of the measurement of the ether drift by the use of an interferometer whose arms are inclined at an angle of forty-five degrees instead of the conventional ninety degrees. The investigation is based upon two hypotheses, both of which are extensively used in explaining the phenomena of physics. The first of these hypotheses is: light is propagated as a wave disturbance, according to Huygen's principle. The second hypothesis is: there exists a stationary medium through which this disturbance known as light is propagated, and with respect to which the earth is moving with some relative velocity.

The principle of the interferometer is well known to physicists. A beam of light striking a half silvered mirror is divided, part of the light being reflected from the plate so as to follow one path, and the other part passing through the plate to a second path. At the ends of their respective paths, each ray is reflected so as to return to the plate, where again one beam is reflected and the other allowed to pass through, and finally both rays are brought to a focus by a lens. Now the division and subsequent reuniting of a beam of light under these conditions creates what are known among physicists as "interference fringes". Also, if the two hypotheses mentioned in the first paragraph are correct,

the location of these fringes should depend slightly upon the orientation of the instrument with respect to the motion of the earth through the ether. The interference fringes themselves are due to the phase relationships between the wave fronts of the reunited rays, and these in turn are affected by the amount and the direction of the motion of various parts of the interferometer itself, during the time while the beam of light is in passage from the plate to the lens. Consequently as the interferometer is turned in different directions these interference fringes will show slight displacements, unless its axis of rotation happens to be perpendicular to the direction of the absolute velocity of the earth.

In carrying out the investigation, the path of each beam of light is carefully traced, from the time it impinges upon the plate, until it enters the lens placed in its final path. Due to the motion of the plate and of each mirror, small changes occur in the angle of reflection, so that it is no longer equal to the angle of incidence, but instead is given by the relation:

$$\tan \frac{\phi'}{2} = \frac{c + u}{c - u} \tan \frac{\phi}{2}$$

where ϕ is the angle of incidence, ϕ' the angle of reflection, c is the velocity of light, and u is the velocity of the reflecting surface in the direction perpendicular to this surface, taken positive when the motion is away from the source.

The first step in the dissertation is the careful calculation of each angle involved in the light paths. During these and in all subsequent calculations, the angle between the principal or source axis of the instrument and the direction of the velocity of the earth, is carried along as a parameter, θ. The expression arrived at as giving the angle between the rays in their final paths is found in equation 26. This equation is:

$$H = \frac{v^2}{c^2}(1.2071067818)(\sin 2\theta + \cos 2\theta)$$

After determining the effect of the motion upon the angles involved in the path, the precise effect of the motion upon the length of each path is found. Corrections are made for the small displacements of the plate and the mirrors while the ray is in passage from one to the other, and the exact length of the path is determined.

The next step taken is the determination of the true wave length of the light along the various paths. The formula used here is the well known relation:

$$\frac{\sin \phi}{\sin \phi'} = \frac{\Sigma}{\Sigma'}$$

where ϕ is the angle of incidence, ϕ' the angle of reflection, Σ is the wave length of the incident light, and Σ' is the wave length of the reflected light. Then, by dividing the wave length along each path into the length of the path, the

total number of wave lengths along each path is found. The number of wave lengths along each path contained between the plate and the lens is then compared, in order to determine the relative lengths of the two paths.

A point very carefully considered is the location of the lens so that the rays will be equally inclined to the axis of the lens. This is accomplished by determining the intersection of the rays in their final paths, and calculating the exact distance along each ray from this point to the plate. Then the shorter of these two distances is increased by an amount sufficient to make the two distances equal, so that if a lens be inserted at this point, or at any point along the final path parallel to this position, the two rays will be equally inclined to its axis.

In making the calculations of these paths and angles, the method of expanding into series is used. It is found that each may be expressed in a power series in $\frac{v}{c}$, where v is the velocity of drift and c is the velocity of light. Since $\frac{v}{c}$ is small, these power series converge very rapidly, in fact, they converge so rapidly that all terms of a higher order than the second are discarded. To this order of approximation, both the sine and the tangent of an angle are equal to the angle itself, a fact frequently made use of.

The final conclusion of the dissertation is arrived at after some forty-seven pages of mathematical deductions. The equation for the difference in the lengths of the paths is

given by equation 106 as:

$$D = \frac{3}{2}\frac{v^2}{c^2}\frac{L}{\lambda}(\sin 2\theta + \cos 2\theta)$$

Now the displacement of the fringe system is related to the path difference in such a way that a change in the length of one path relative to the other of an amount equal to one wave length, causes a shift equal to the width of one fringe. Hence, the above formula gives the displacement of the fringe system from a zero position, as the interferometer is rotated to make different angles with the direction of drift. A study of the above equation shows, then, that the total fringe shift which may be expected from an interferometer with arms inclined at forty-five degrees, is:

$$F = 3\sqrt{2}\,\frac{v^2}{c^2}\frac{L}{\lambda}$$

Comparing this with the total fringe shift obtainable from a rectangular interferometer, namely:

$$F' = 2\,\frac{v^2}{c^2}\frac{L}{\lambda}$$

we see that the expected fringe shift is more than doubled by this simple change in the construction.

Appendix A

PROOF OF EQUATION 3, THE LAW OF REFLECTION FROM A MOVING SURFACE.

The left hand sketch on plate 8-A is to be used in connection with this discussion. AA'A" represents the surface of a mirror moving in the direction v as shown by the small velocity diagram. Let ϕ = the angle of incidence. While the ray of light travels from B to B', the mirror moves from A' to B'. Let the common time required be t. Suppose AB to represent a wave front traveling as shown. By the time point B of the wave front overtakes the mirror at B', the points between A and B have already been reflected from the mirror, giving rise (according to Huygen's principle) to a series of circular expanding waves which blend in a manner to form the reflected wave front B"B'. In other words, while the incident ray of light travels from B to B', the reflected ray travels from A to B", and similarly for all points between A and B. Then:

$$BB' = ct$$

$$A'B' = vt$$

$$A''B' = ut$$

In the right triangles ABB' and AB"B'

AB' is common to each and AB" = BB'

Therefore: $\phi + a^{(1)} = \phi' - a$

- -

(1) a has been substituted for the alpha in the figure.

so: $a = \frac{\phi' - \phi}{2}$

and $\sin a = \sin \frac{\phi' - \phi}{2} = \frac{A''B'}{AB'}$

$$\phi + a = \phi + \frac{\phi' - \phi}{2} \qquad \frac{\phi' + \phi}{2}$$

Therefore: $\sin(\phi + a) = \sin \frac{\phi' + \phi}{2} = \frac{BB'}{AB'}$

Hence: $$\frac{\sin \frac{\phi' + \phi}{2}}{\sin \frac{\phi' - \phi}{2}} = \frac{BB'}{A''B'} = \frac{ct}{ut} = \frac{c}{u}$$

$$\frac{\sin(\frac{\phi'}{2} + \frac{\phi}{2}) - \sin(\frac{\phi'}{2} - \frac{\phi}{2})}{\sin(\frac{\phi'}{2} + \frac{\phi}{2}) + \sin(\frac{\phi'}{2} - \frac{\phi}{2})} = \frac{c - u}{c + u}$$

Expanding the left hand member:

$$\frac{\cos \frac{\phi'}{2} \sin \frac{\phi}{2}}{\sin \frac{\phi'}{2} \cos \frac{\phi}{2}} = \frac{c - u}{c + u} = \frac{\tan \frac{\phi}{2}}{\tan \frac{\phi'}{2}}$$

And therefore: $\tan \frac{\phi'}{2} = \frac{c + u}{c - u} \tan \frac{\phi}{2}$ Q. E. D.

Here the direction of drift has been assumed away from the incident light, hence v is posituve when the mirror moves away from the source, and negative when the plate moves toward the source, of incident light.

Appendix B

DERIVATION OF EQUATION 10 FROM EQUATION 9

Equation 9 is:

$$\tan\frac{\phi_1'}{2} = \tan\frac{3}{16}\pi + \frac{v}{c}\left[2\tan\frac{3}{16}\pi\sin\left(\theta+\frac{\pi}{8}\right) - \frac{1}{2}\sec^2\frac{3}{16}\pi\sin\theta\right]$$

$$+\frac{v^2}{c^2}\left[2\tan\frac{3}{16}\pi\sin^2\left(\theta+\frac{\pi}{8}\right) + \frac{1}{4}\tan\frac{3}{16}\pi - \sec^2\frac{3}{16}\pi\sin^2\theta -\right.$$

$$\left.\sin\theta\sin\left(\theta+\frac{\pi}{8}\right)\sec^2\frac{3}{16}\pi\right]$$

Treating the coefficient of $\frac{v}{c}$ in the above, by expanding out $\sin(\theta+\frac{\pi}{8})$ and substituting the values of known constants, we find it becomes: $\frac{v}{c}[.5114017893(\sin\theta+\cos\theta)]$

The coefficient of $\frac{v^2}{c^2}$ in equation 9 may also be expanded and evaluated for known constants, with the result:

$$\frac{v^2}{c^2}[.0459188760\sin^2\theta + .3914099850\sin\theta\cos\theta + .1957049920\cos^2\theta]$$

Hence, substituting the numerical value of $\tan\frac{3}{16}\pi$, we get for equation 10 the result:

$$\tan\frac{\phi_1'}{2} = .6681786378 + \frac{v}{c}[.5114017893(\sin\theta+\cos\theta)] +$$

$$\frac{v^2}{c^2}[.0459188760\sin^2\theta + .3914099850\sin\theta\cos\theta +$$

$$.1957049920\cos^2\theta]$$

Appendix C

CHANGE IN WAVE LENGTH ACCOMPANYING REFLECTION FROM A MOVING SURFACE.

Due to the fact that when light is reflected from a moving surface the angle of reflection is not exactly equal to the angle of incidence, there is an accompanying change in wave length. The right hand figure shown on plate 8-A illustrates the geometric picture, and from the results shown: $\lambda_1 = AB \sin \phi$, $\lambda_2 = AB \sin \phi'$, we draw the conclusion that:

$$\frac{\lambda_1}{\lambda_2} = \frac{\sin \phi}{\sin \phi'}$$

where λ_1 and λ_2 refer to the wave lengths of the incident and reflected light, respectively, and ϕ and ϕ', to the angles of incidence and reflection.

Appendix D

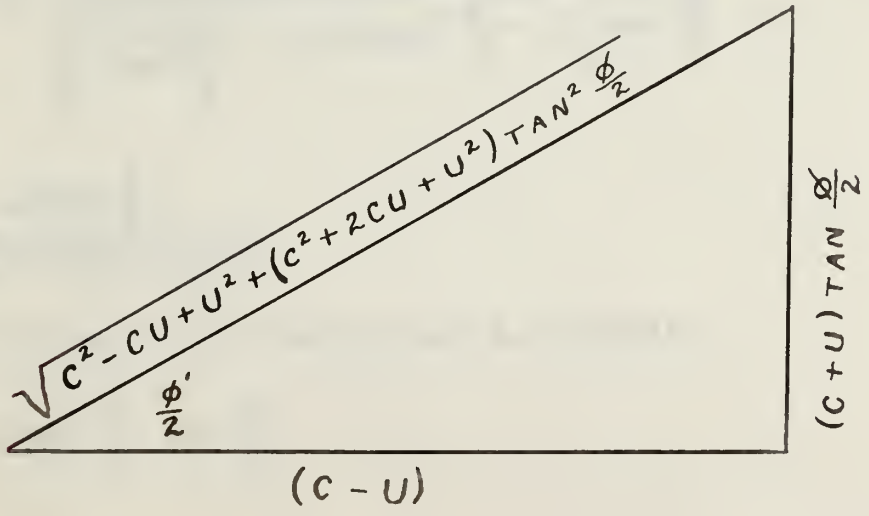
DERIVATION OF THE EQUATION GIVING THE REFLECTED WAVE LENGTH AS A FUNCTION OF THE VELOCITIES AND THE ANGLE OF INCIDENCE

(Equation no. 87)

If ϕ is the angle of incidence, ϕ' the angle of reflection, c = the velocity of light, u = velocity of reflecting surface in the direction perpendicular to itself, positive when the surface moves away from the source, Σ = the wave length of the incident light, and Σ' the wave length of the reflected light, the following equations have been derived in parts A and C of the appendix:

$$\tan \frac{\phi'}{2} = \frac{c + u}{c - u} \tan \frac{\phi}{2} \qquad (1)$$

$$\frac{\Sigma'}{\Sigma} = \frac{\sin \phi'}{\sin \phi} \qquad (2)$$



It is evident that the hypotenuse above may be written:

$$\sqrt{c^2(1 + \tan^2 \frac{\phi}{2}) + u^2(1 + \tan^2\frac{\phi}{2}) + 2cu(\tan^2\frac{\phi}{2} - 1)}$$

$$\pm\sqrt{\frac{c^2 + u^2}{\cos^2 \frac{\phi}{2}} + 2cu(\tan^2 \frac{\phi}{2} - 1)}$$

Hence: $\sin \frac{\phi'}{2} = \frac{(c + u)}{\sqrt{\frac{c^2 + u^2}{\cos^2 \frac{\phi}{2}} + 2cu(\tan^2 \frac{\phi}{2} - 1)}} \frac{\sin \frac{\phi}{2}}{\cos \frac{\phi}{2}}$

$$= \frac{(c + u)\sin \frac{\phi}{2}}{\sqrt{c^2 + u^2 + 2cu(\sin^2 \frac{\phi}{2} - \cos^2 \frac{\phi}{2})}}$$

$$= \frac{(c + u) \sin \frac{\phi}{2}}{\sqrt{c^2 + u^2 - 2cu\cos \phi}}$$

Also: $\cos \frac{\phi'}{2} = \frac{(c - u)}{\sqrt{\frac{c^2 + u^2}{\cos^2 \frac{\phi}{2}} + 2cu(\tan^2 \frac{\phi}{2} - 1)}} \frac{\cos \frac{\phi}{2}}{\cos \frac{\phi}{2}}$

$$= \frac{(c - u)\cos \frac{\phi}{2}}{\sqrt{c^2 + u^2 - 2cu \cos \phi}}$$

Now substituting in the trigonometric identity:

$$\sin \phi' = 2 \sin \frac{\phi'}{2} \cos \frac{\phi'}{2}$$

$$= \frac{2(c^2 - u^2)\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{c^2 + u^2 - 2cu\cos\phi}$$

$$= \frac{(c^2 - u^2)\sin\phi}{c^2 + u^2 - 2cu\cos\phi}$$

or, we may write:

$$\frac{\sin\phi'}{\sin\phi} = \frac{c^2 - u^2}{c^2 + u^2 - 2cu\cos\phi} \qquad (3)$$

and by equation 2:

$$\frac{\Sigma'}{\Sigma} = \frac{c^2 - u^2}{c^2 + u^2 - 2cu\cos\phi}$$

From this follows equation no. 87 in the main body of the thesis:

$$\Sigma' = \Sigma \frac{(c^2 - u^2)}{c^2 + u^2 - 2cu\cos\phi}$$

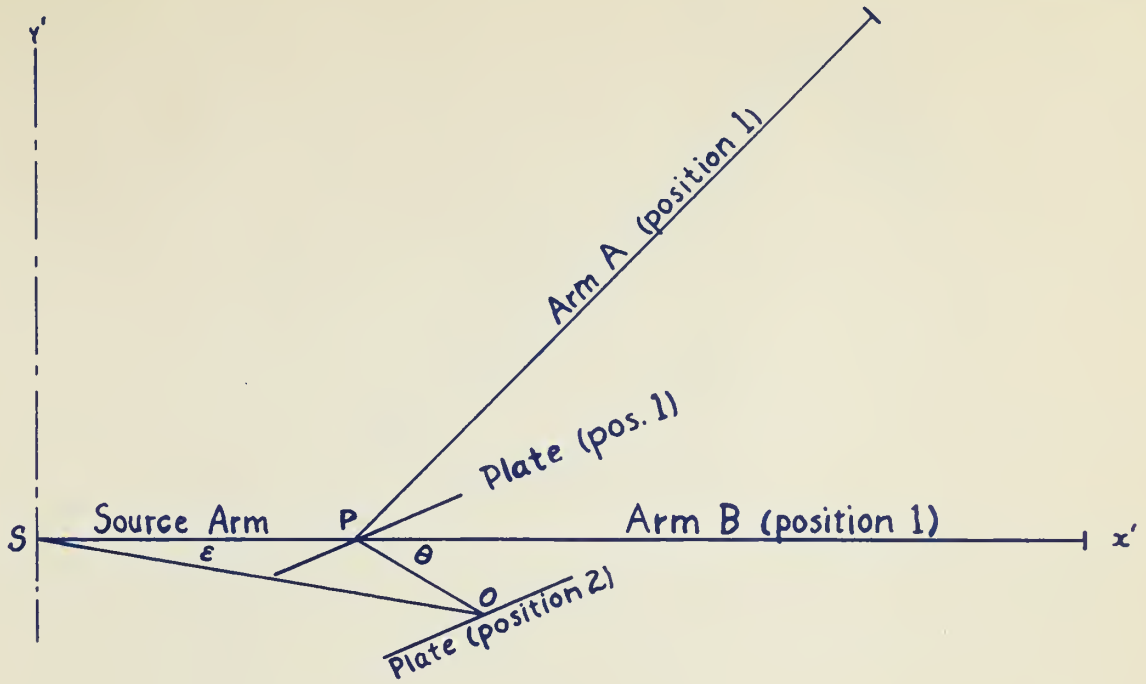


PLATE 1.

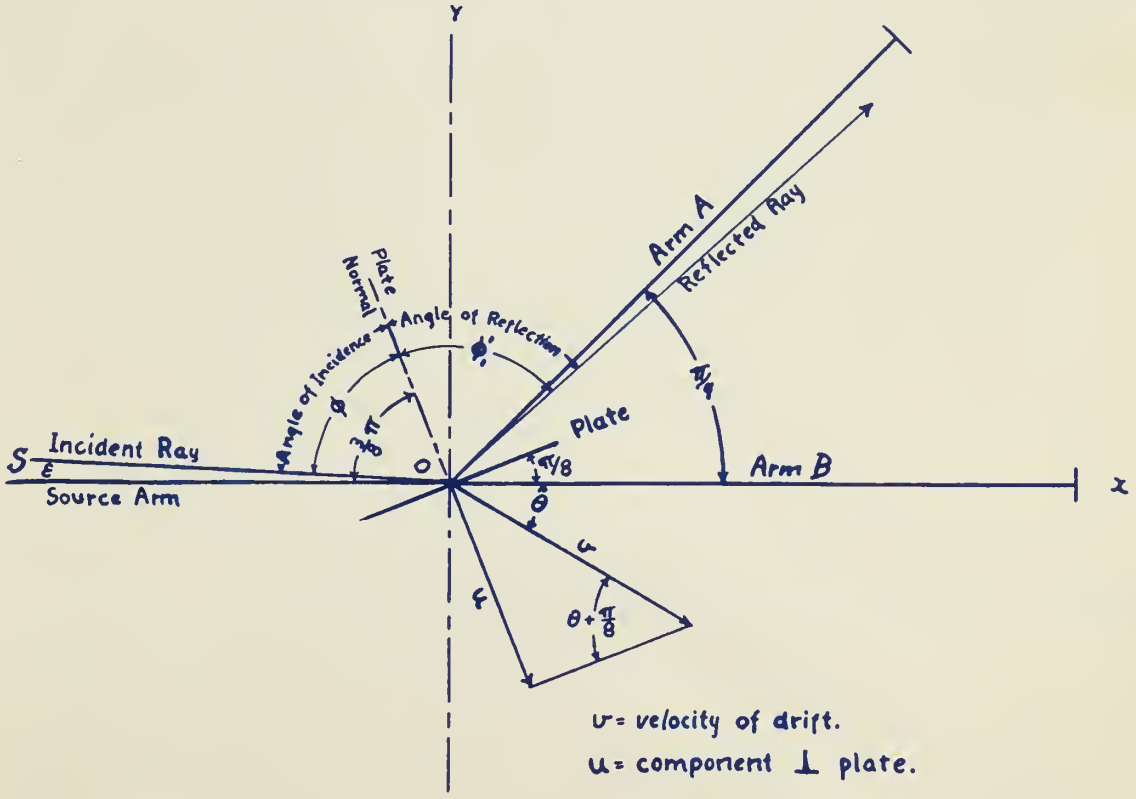


PLATE 2.

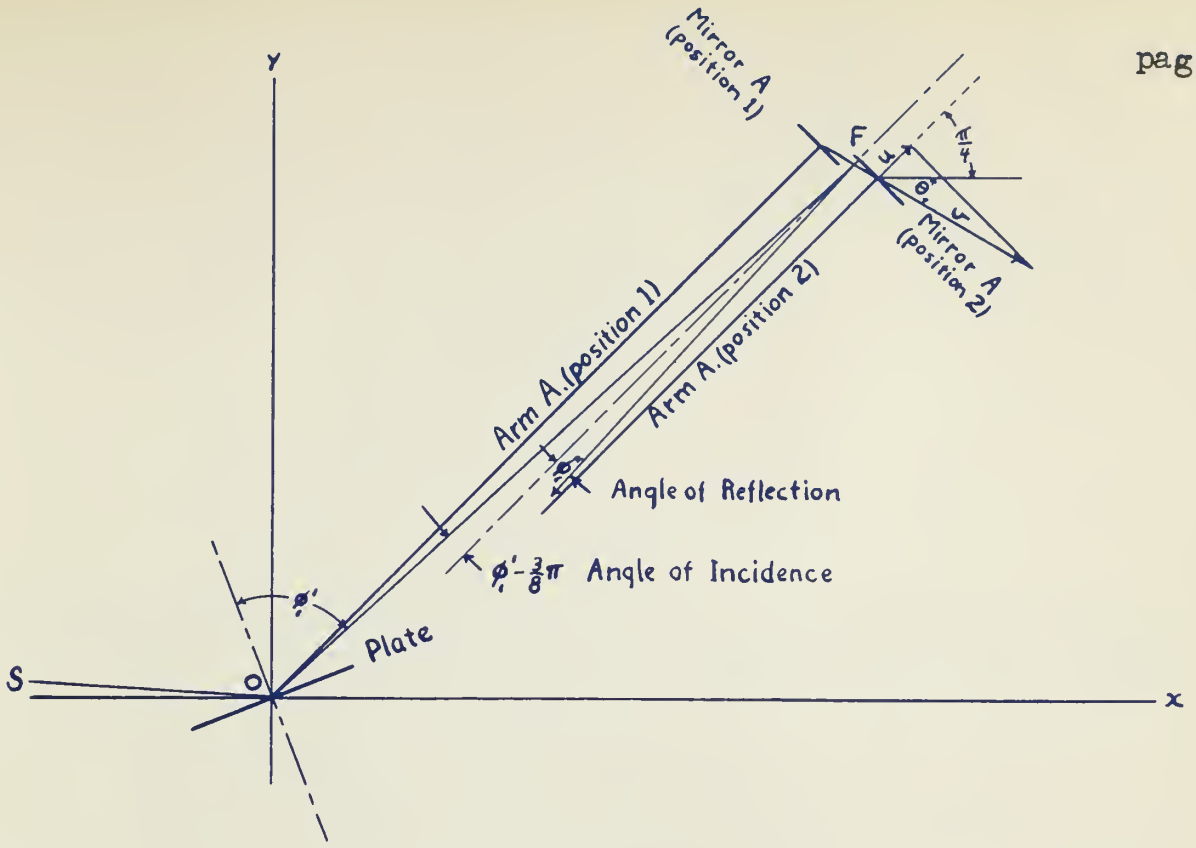


PLATE 3.

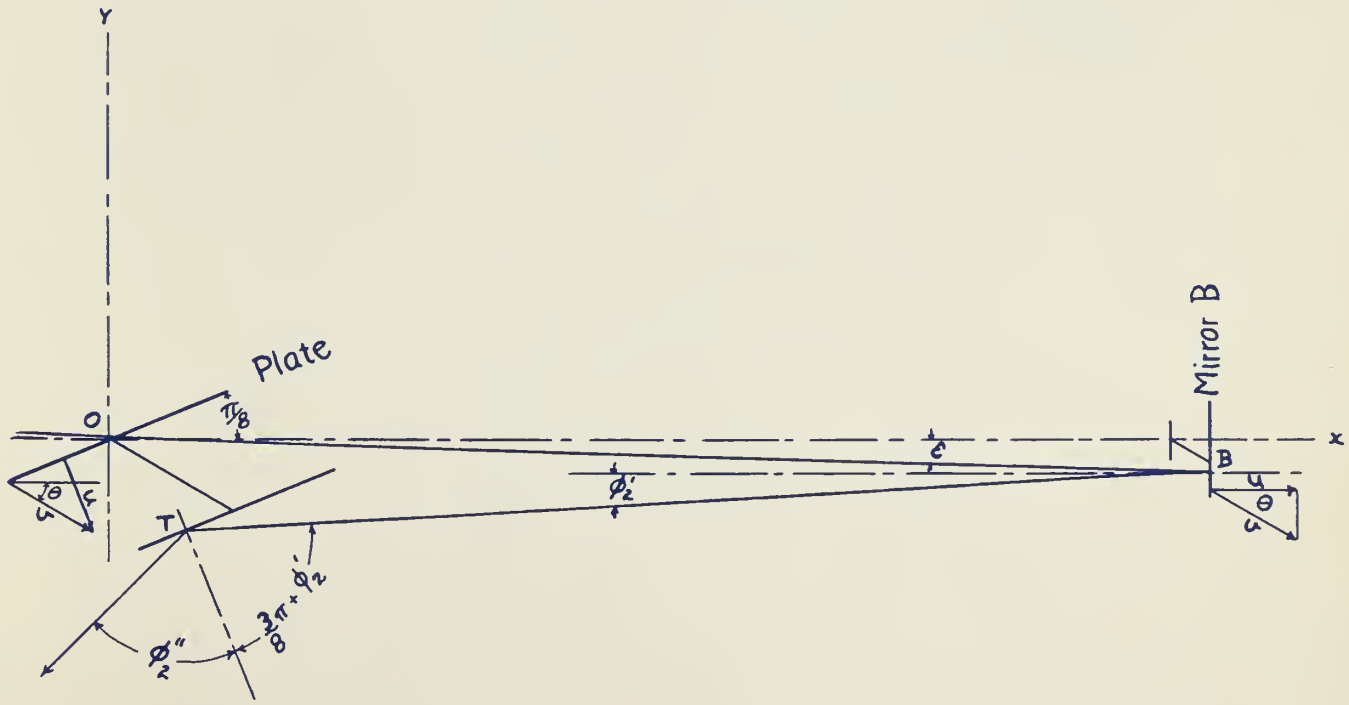


PLATE 4

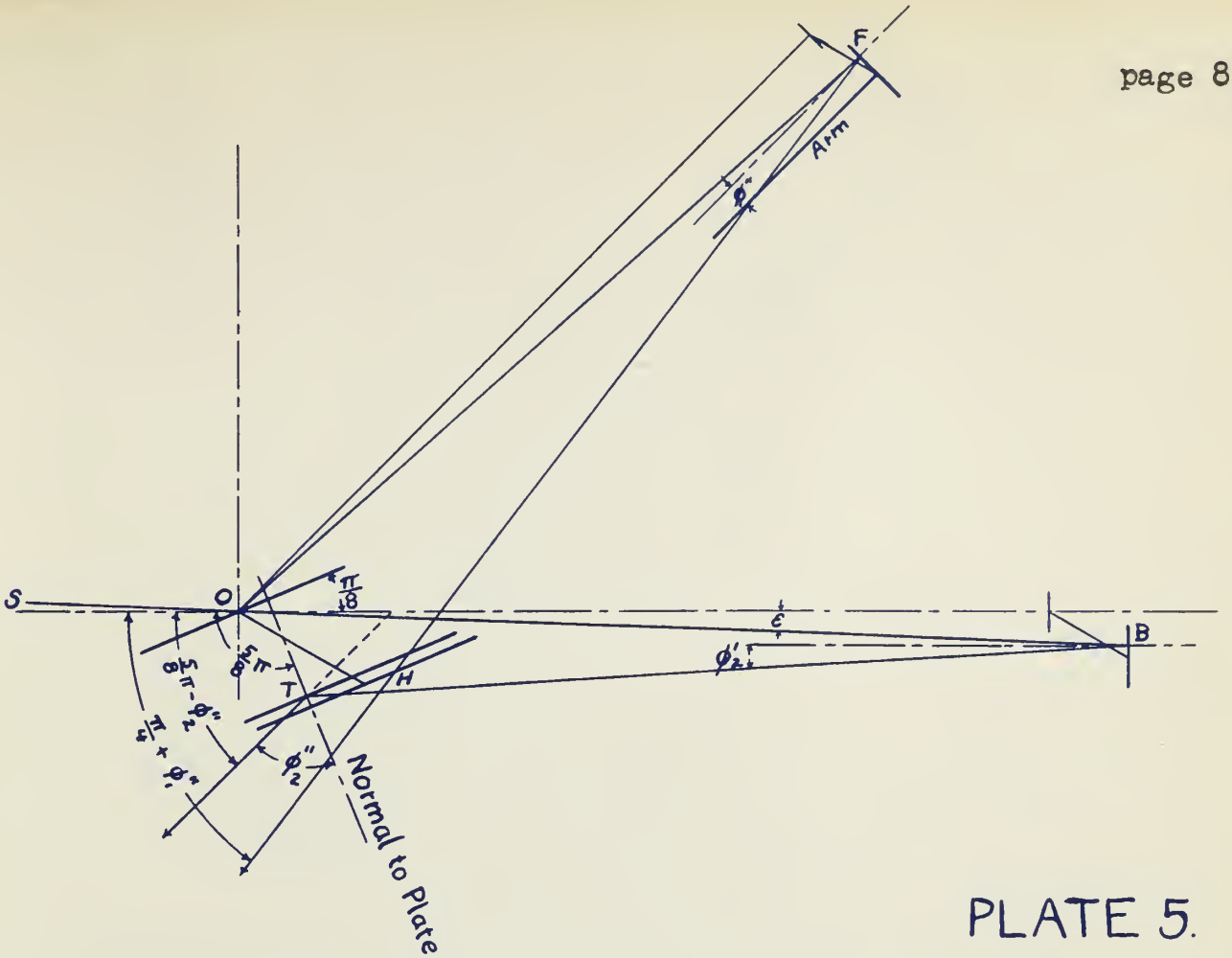


PLATE 5.

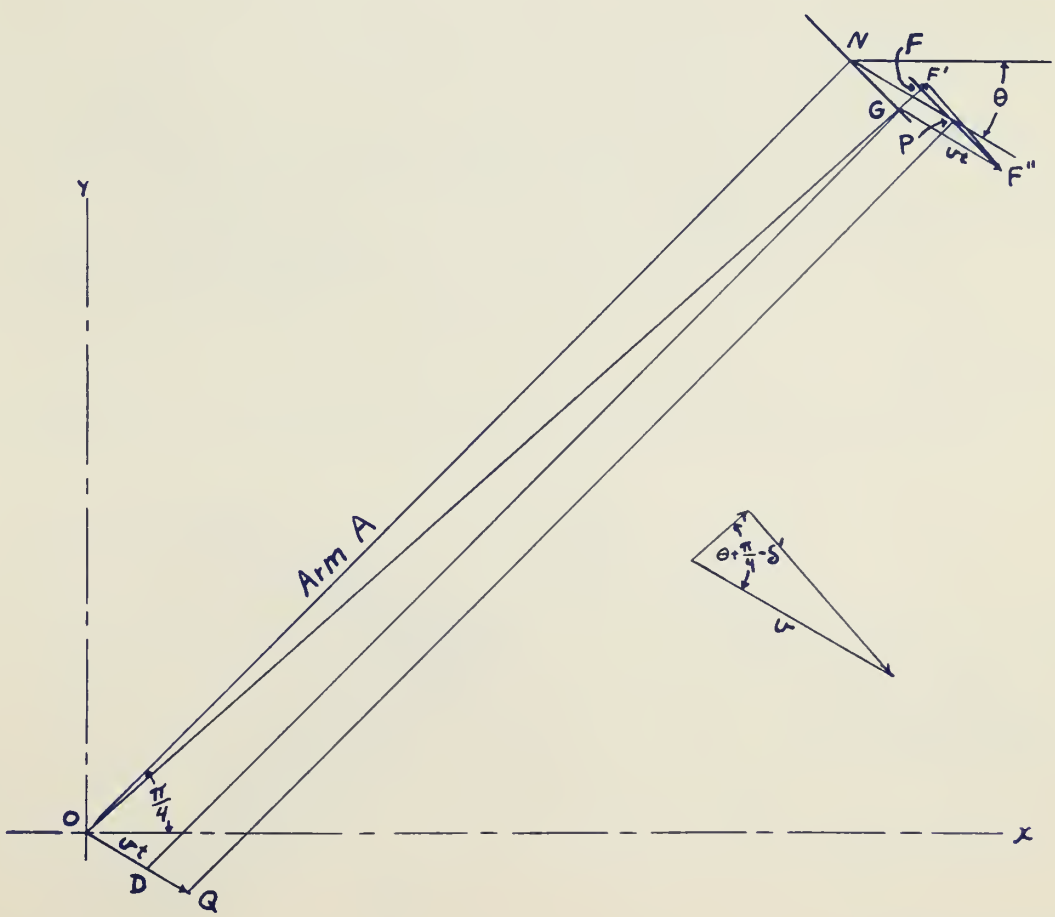


PLATE 6.

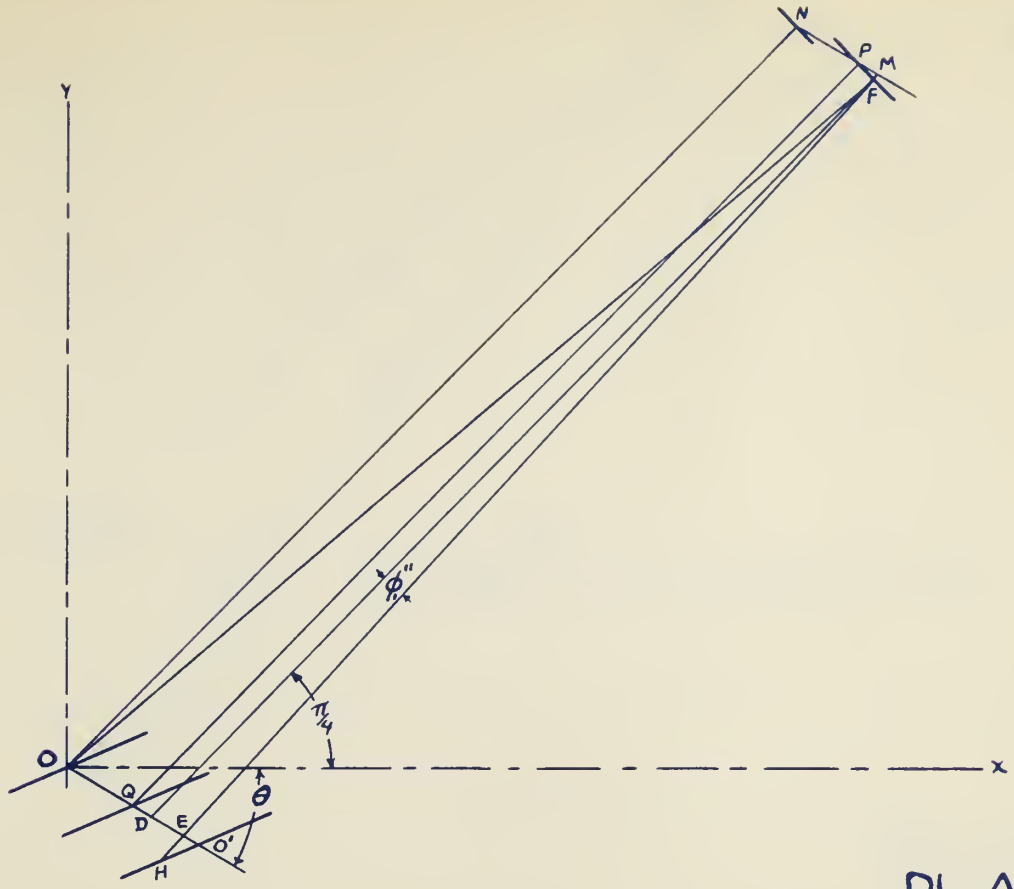


PLATE 7

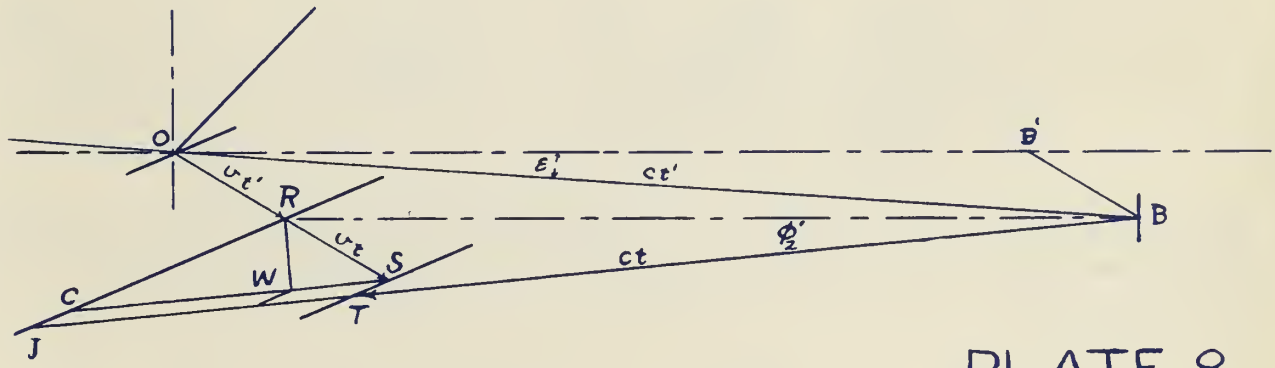
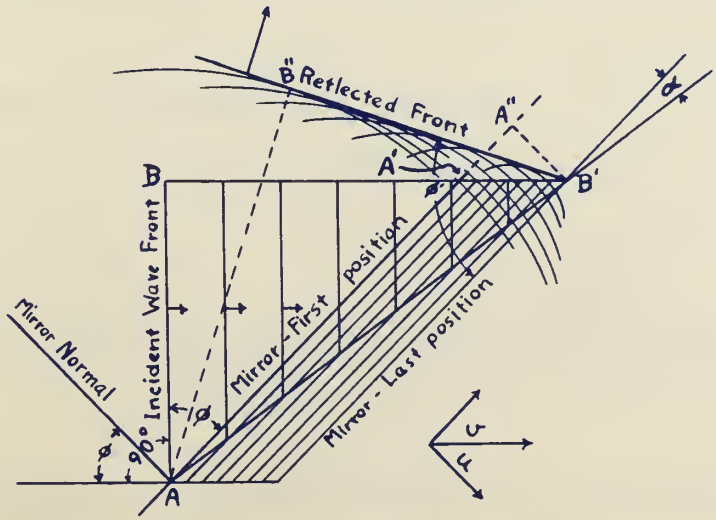
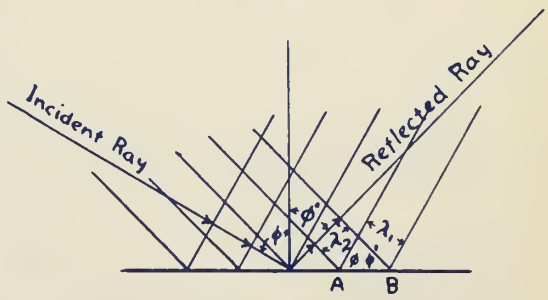


PLATE 8.





$\lambda_1 = AB \sin \phi$

$\lambda_2 = AB \sin \phi'$

PLATE 8A

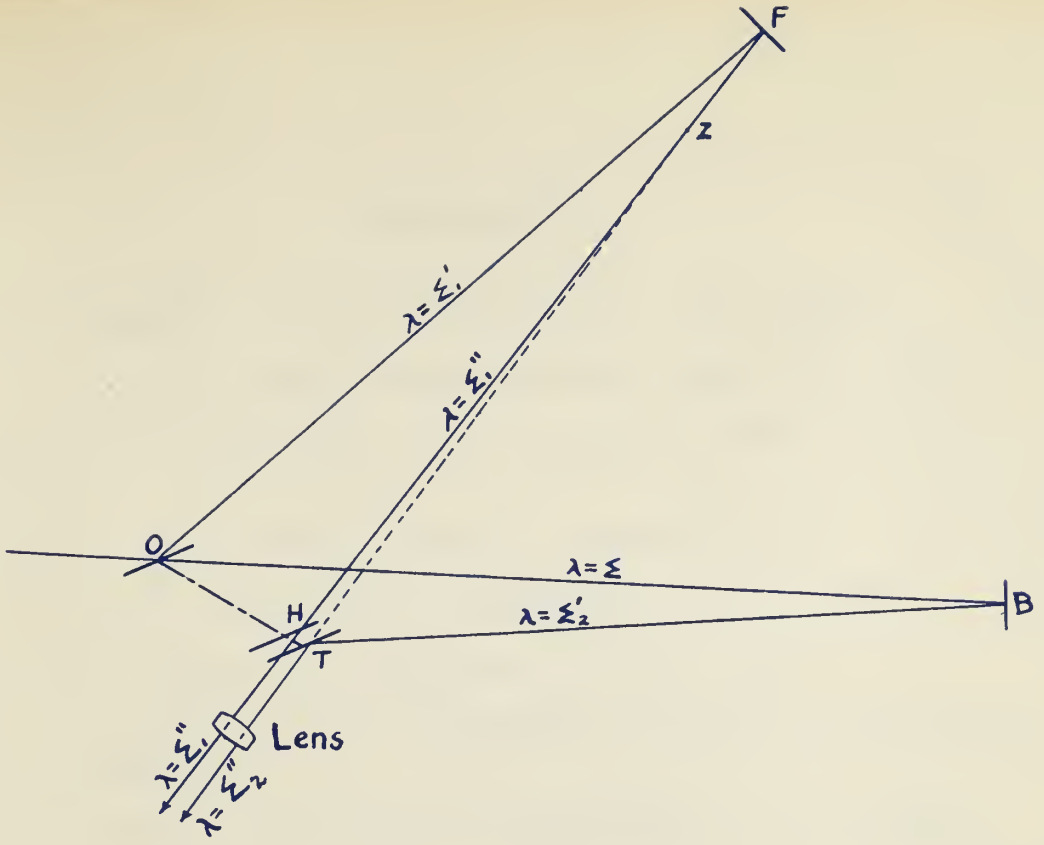
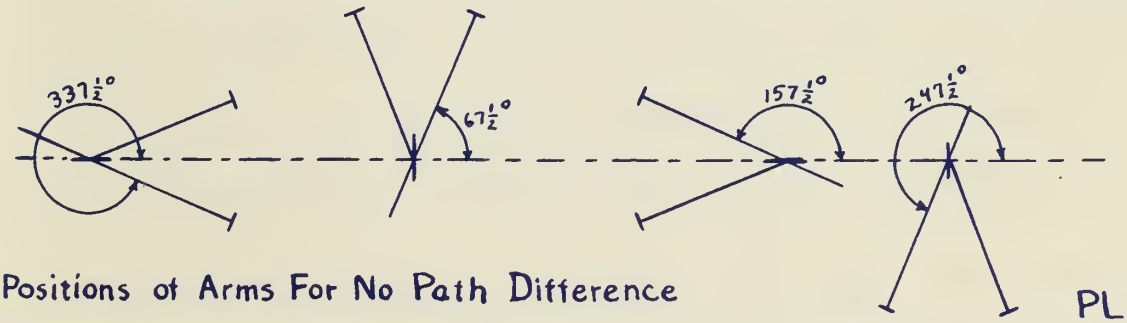


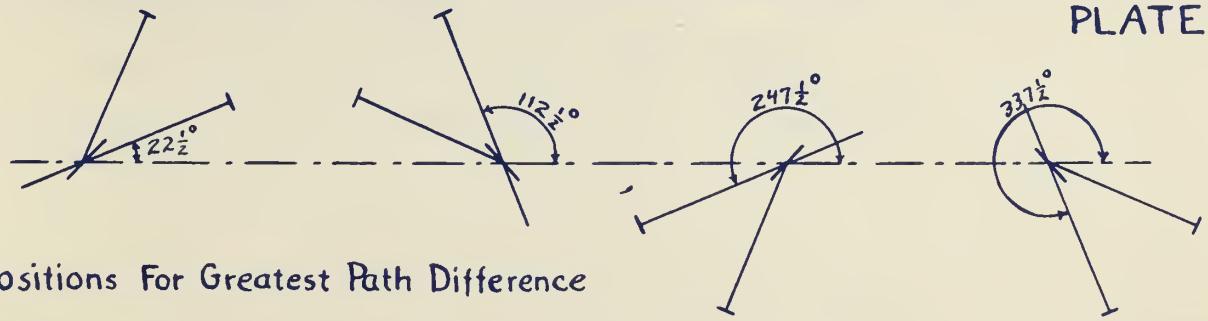
PLATE 9



Positions of Arms For No Path Difference

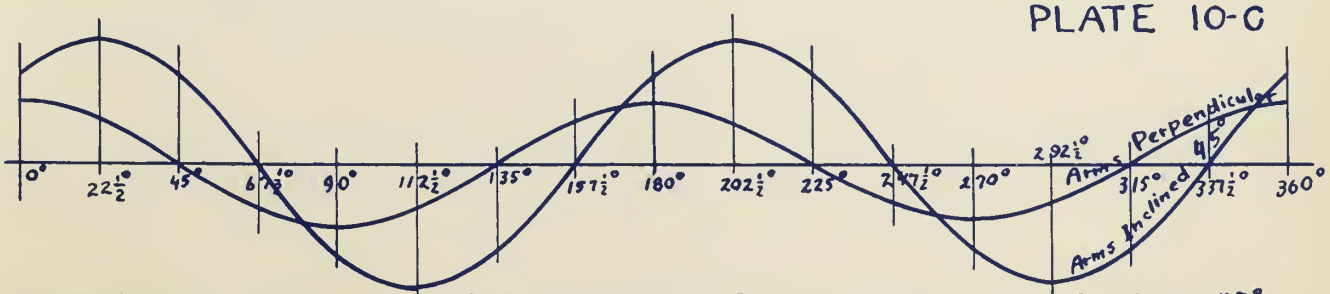
PLATE 10-A

PLATE 10-B



Positions For Greatest Path Difference

PLATE 10-C



Comparison of Expected Displacements–Arms Perpendicular and Arms Inclined 45°

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AUTOBIOGRAPHY

The author was born August 1, 1904, in Dighton, Massachusetts, the son of Winifred Slade and Eugene Brightman Haskins. He was graduated from the country grammar school in Dighton in 1917, and from Fall River High School in 1921. In 1925 he received the degree Bachelor of Mechanical Engineering from Northeastern University.

During the school year 1925-1926 he taught in the high school at Fairchance, Pennsylvania, and during the four years following, at the high school in Monongahela, Pennsylvania. The summers of 1926 and 1927 were spent in study at the University of West Virginia, courses being taken in mathematics and education. During the four years while he resided at Monongahela, he took courses at the University of Pittsburgh, receiving from that institution the degree of Master of Arts in June, 1930.

In September, 1930, he was appointed Instructor of Mathematics at Northeastern University where he has continued until this time. He was made Assistant Professor of Mathematics in September, 1936.

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